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## The Significance of Symbols in Physical Equations


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THE quantitative definitions and laws of physics are expressed by equations such as $v=s/t$ and $F=ma$. In this paper I shall discuss the significance of the symbols in these equations. That the problem of determining what the symbols designate is real, is indicated by the existence of several solutions. The view expressed by Bridgman is that the symbols denote numbers which are the results of measurement. J. Wallot represents those who contend that the symbols may denote the physical quantities directly. The need for clarity in one's point of view is demonstrated by the following quotation from A. G. Worthing's review of a popular textbook. He says,

> There is failure to follow a consistent policy with regard to the significance of symbols that are used in the equations which occur. Consistently they may be used to represent physical quantities or plain numerics. Judged by his usage the author very much prefers to have the symbols represent physical quantities, a policy which most teachers of physics will heartily endorse. That policy, however, has not been strictly adhered to in this book. Too often the symbol stands for a numeric, and what is particularly bad, both usages are actually to be found in several individual equations.

It is desirable to give a preliminary definition of the term *quantity*. According to Bertrand Russell, a quantity is a particular object such as a meterstick, the length of which is a magnitude. Two equal quantities have the same magnitude. For example, two metersticks have the same length and are therefore equal in respect to length. Magnitudes thus are more abstract than quantities. The distinction between quantity and magnitude appears to be useful, for we speak of a quantity of water as a particular object that is characterized by a magnitude mass. In contemporary discussion, however, the word *quantity* is also used in the sense of magnitude. Measurable properties, such as length, speed and mass, are called quantities. To add to the ambiguity, Eddington applies the word *quantity* to numerical measures. In the present paper I shall conform to contemporary usage and treat physical quantities as measurable properties or magnitudes.

The result of a measurement of a given quantity is expressed by a number relative to a unit of the quantity. For example, if $l$ designates a particular length, the result of measuring it may be expressed by $(l)[L]$, where $(l)$ denotes the numerical measure and $[L]$ the unit of length. The issue in the discussion is: Do the symbols in an equation designate quantities (measurable properties) directly or do they represent numerical measures? In particular, if the symbol $l$ occurs, does it designate the physical property of a line, or does it denote the numerical measure $(l)$?

### Symbols as designating numbers

The operational theory of physical concepts appears to demand the interpretation of symbols in terms of numbers. A length occurs in physics through the numerical measure obtained by operations. Indeed, the operationalist applies the word *quantity* to the numerical measure as alone



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having physical significance. Furthermore, it has been argued that the mathematical operations of multiplication, division and so forth apply only to numbers and hence that an equation such as $v=s/t$ is meaningless unless the symbols designate numbers. It may be remarked that, if one interprets the symbol $l$ as standing for a number, one should not write $l=12$ cm but should use the form, "$12=l$ in centimeters."

**Symbols as designating quantities**

Despite the theoretical arguments in favor of the foregoing view, widespread use is being made of the interpretation of symbols in terms of quantities. The principal purpose of this paper is to examine the justification for this interpretation. By way of introduction, I remark that a physical quantity may be given a physical status by viewing it as a relation—for example, between a given line and a unit of length.

A defense of the interpretation of symbols in terms of quantities appears to require an answer to the criticism that mathematical operations apply only to numbers. Now, mathematical operations may be defined for any objects whatsoever. For example, vector and scalar multiplication operate upon vectors that are directed segments of straight lines. The possibility of defining operations upon quantities may be demonstrated by building a miniature physical theory of length. We may begin with the empirical fact that in nature there exist practically straight lines having the physical property *length*. We may represent the length of a line by the symbol $l$. Two lengths $l_1$ and $l_2$ can be added and the sum denoted by $l_1+l_2$. We may show experimentally that $l_1+l_2=l_2+l_1$, the commutative law for addition. We may use equal lines to form a square and define its area as denoted by $l^2$. We may form a cube and define its volume as denoted by $l^3$. We may next form the symbol $l^4$, but there then arises the problem of the status of the object it designates. I think that the formation of $l^4$ indicates that we have passed from operations on objects to operations on symbols. Again, if the symbol $t$ designates a lapse of time, what physical object is designated by $t^2$?

In order to explain further the possibility of defining operations upon quantities, let us consider the equation $v=s/t$. What is the meaning of the sign of division? Following Wallot, we may say: If the distance traveled in a given time is $n$ times greater, the quantity $v$ becomes $n$ times larger. If the time required to travel a given distance is $n$ times greater, the quantity $v$ becomes $n$ times smaller. It may be contended, however, that this explanation of the operation of division for quantities is based upon numerical measures.

Although we can give interpretations to operations upon quantities, the mathematical development immediately leads to the interpretation of the operations as upon the symbols for the quantities. The symbolic interpretation of operations is strengthened by considering the significance of the symbol for equality. The equation $A=l^2$, where $A$ stands for area, may express the fact that $A$ is analyzed in terms of $l$. But consider $F=ma$. The symbols $m$ and $a$ symbolize properties of a body, whereas $F$ symbolizes conditions in the environment. For example, if a force exerted by a spring acts on a body, then $F=-kx$, where $k$ and $x$ refer to properties of the spring. Thus, the symbol for equality in $F=ma$ does not stand for an objective relation. $F=ma$ is not an analysis of $F$ in the same way that $A=l^2$ is an analysis of $A$. $F=ma$ stands for $F=kma$ with $k=1$, and this in turn means that $F$ is directly proportional to mass and acceleration conjointly. Our interpretation of the symbol for equality should be that for the symbol $F$ we may substitute $ma$ in a calculus of symbols. Thus ultimately our symbols for operations apply to symbols and not to the quantities directly.

**Symbols as designating labeled numerals**

The conclusion that symbols of operations upon quantities are finally to be viewed as operating on symbols suggests that we may mediate between the view that symbols designate numbers and the view that symbols designate quantities by Helmholtz' doctrine that a symbol designates a *benannte zahl*. This doctrine may be set forth for the physical quantity length which has been represented by the symbol $l$. Now a length may be viewed as the sum of a set of equal units of length. This analysis expresses the fact that, in measuring a length, one counts the number of times that the unit can be laid off on the length. A sum of equal units may be expressed

as the product of the cardinal number of the set and the unit. Accordingly, the result of measuring a length is appropriately described by the symbol $(l)\ [L]$, where $(l)$ symbolizes the number and $[L]$ the unit of length. We may write $l=(l)[L]$, where the symbol for equality expresses an objective relation. The symbol $(l)[L]$, which describes the result of measurement and which I shall call a *labeled numeral*, expresses the product of a number and a unit. The function of measurement is to describe a quantity as such a product. The product may be called a *concrete number*, and hence the symbols in physical equations designate labeled numerals which express quantities as concrete numbers.

In Helmholtz' writings a *benannte zahl* is sometimes said to be formed as the sum of a cardinal number of equal quantities; sometimes a quantity is said to be expressed through a *benannte zahl*. Helmholtz interprets a number as a symbol. The apparent unclearness in his doctrine can be overcome by making a distinction between a number and its symbol, the *numeral*. In this paper a labeled numeral is the symbol of the product of a cardinal number and a unit.

The product, which I call a *concrete number*, is embodied in a quantity as a sum of equal units. The quantity is expressed through the labeled numeral which symbolizes the quantity grasped as a concrete number. Thus the symbols in physical equations designate labeled numerals which in turn express physical quantities. The unlabeled symbols may be called *unanalyzed* labeled numerals.

Our discussion has been based upon the quantity *length* which is readily analyzed into a sum of equal units. In addition to such extensive quantities, the development of physics requires the introduction of intensive quantities such as temperature. Intensive quantities, however, may be handled as if they were the products of numbers and units. Hence it is a general doctrine that the symbols in equations may designate labeled numerals, each of which expresses the product of a number and a unit.

**Applications**

In the light of the foregoing discussion, it appears that physical equations should be formed so that the symbols are unanalyzed labeled numerals. Such equations will be independent of the choice of units. Foundations for this point of view have been created by Wallot with his doctrine of equations for quantities. In applying the equations one may substitute labeled numerals for the unlabeled symbols; thus the numerical value is provided with the unit to which it is relative. The symbols for operations upon unlabeled symbols thus become symbols for operations upon labeled numerals. The labels for the units are subject to the same mathematical operations as the numerals. There arise equations in the labels of the units which must be interpreted. The possibility of interpretation is founded upon that of equations in which the symbols designate quantities.

For example, let us analyze $v=(v)[V]$, where $v$ stands for speed. Let the defining equation for $v$ be $v=s/t$.
Then

$$(v)[V]=(s)[L]/(t)[T].$$

We then obtain

$$[V]=\frac{(s)}{(v)(t)}\frac{[L]}{[T]},$$

an equation that expresses the unit $[V]$ in terms of the units $[L]$ and $[T]$. Suppose $[V]=[L]/[T]$, which means that unit speed is defined as the speed with which unit distance is traveled in unit time. Then

$$\frac{(s)}{(v)(t)}=1, \quad \text{or} \quad (v)=\frac{(s)}{(t)}.$$

Thus, if we define speed by $v=s/t$, and define the unit of speed in terms of the units of length and time by $[V]=[L]/[T]$, then $(v)=(s)/(t)$. If such adjusted units are adopted, the equation for the measures is the same as the equation for the quantities. But we may decide to define $[V]$ by $[V]=k[L]/[T]$. Then $(s)/(v)(t)=k$, and

$$(v)=\frac{1}{k}\frac{(s)}{(t)}.$$

For example, let $[L]=$centimeter, $[T]=$second and $[V]=$meter per second. Then $[V]=100$ cm/sec, and hence

$$(v) \text{ in meters per second}=\frac{1}{100}\frac{(s) \text{ in cm}}{(t) \text{ in sec}}.$$

Let us apply the foregoing procedure to the force equation $F=ma$. We may write $(f)[F]=(m)[M](a)[A]$.
Then

$$[F]=\frac{(m)(a)}{(f)}[M][A].$$

If we define $[F]=[M][A]$, then $(m)(a)/(f)=1$ and $(f)=(m)(a)$. For example, if dyne = gram cm/sec², then $(f)$ in dynes = $(m)$ in grams × $(a)$ in cm/sec². Suppose that the gram weight is defined as a unit of force by the equation

gram weight = 980 grams cm/sec²
= gram 980 cm/sec².

Then $(m)(a)/(f)=980$, and $(f)=(m)(a)/980$.

The employment of the preceding method has been attributed to Stroud in England. It has been advocated by D. L. Webster and is partly used in some American textbooks. The most careful discussion has been given by Wallot, and in Germany a committee consisting of J. Wallot, F. Emde, H. Diesselhorst, G. Hamel, W. Kösters, G. Madelung, H. Reissner and K. Scheel has explained its application. The primary function of the present paper has been to analyze the theoretical basis of the procedure.


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## Field Vectors and Unit Systems in the Theory of Electricity


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RECENT discussions of mks electric units have convinced the writer that physical relations have become obscured by symbolism; that it is time to get back to essentials by stripping the equations of everything that is merely traditional or elegant. This article is an attempt in that direction, with one object particularly in view: to put the theory into a general form in which the units are arbitrary and from which the special forms for the systems of units in common use can be obtained as simply as possible. In keeping with the general aim of sticking to essentials, the scalar and vector potentials will be omitted, and no symbols will be introduced until the quantities they represent occur in the natural development of the theory.

No apology is needed nowadays for basing magnetostatics on the law of force between currents rather than on the law of force between magnets. No apology should be needed, either, for discarding the magical properties of the "medium." These properties, so far as they really exist, can and should be derived from the properties of the atoms of which the "medium" consists.

### §1. THE MICROSCOPIC EQUATIONS[1]

We begin with the electrostatic force exerted by a stationary particle of charge $q'$ on a stationary particle of charge $q$,

$$\mathbf{F}_e=k'qq'\mathbf{r}_0/r^2, \tag{1a}$$

and the magnetostatic force exerted by a slowly moving charge $q'$ on a slowly moving charge $q$,

$$\mathbf{F}_m=kq\mathbf{v}\times(q'\mathbf{v}'\times\mathbf{r}_0/r^2), \tag{1b}$$

where $\mathbf{r}$ is the vector from $q'$ to $q$ (of magnitude $r$ along unit vector $\mathbf{r}_0$), the $\mathbf{v}$'s are the velocities of the two charges and the $k$'s are constants of proportionality. The mechanical units and the unit of charge are arbitrary. The $k$'s will have values that depend on the choice of units; but since the $q$'s occur to the same power in both equations, the ratio $k'/k$ is dependent only on the mechanical units and independent of the

[1] Some of the points covered here in regard to units have been discussed in detail by Warburton, Am. J. Phys. **8**, 30 (1940). The presentation follows that of Mason and Weaver, *The electromagnetic field* (Univ. Chicago Press, 1929), except that they leave the time averaging until later.

unit of charge. In fact

$$k'/k=c^2, \tag{2}$$

where $c$ is a quantity with the dimensions of velocity. Experimentally $c\doteq 3\times10^8$ m/sec, a fact that we are entitled to state without having to choose a unit of charge.

The total force on $q$ is to be found by summing Eq. (1a) or (1b) over all charges $q'$. This done, we find it convenient to separate the force into a factor depending on the *test charge* $q$ and a factor depending on the charges $q'$ and on $\mathbf{r}$; for instance,

$$\mathbf{F}_e=q\mathbf{E}, \quad \mathbf{F}_m=q\mathbf{v}\times\mathbf{B}, \tag{3}$$

where the electric and magnetic *field vectors* $\mathbf{E}$ and $\mathbf{B}$ are defined by

$$\mathbf{E}=k'\sum q'\mathbf{r}_0/r^2, \tag{4a}$$

$$\mathbf{B}=k\sum q'\mathbf{v}'\times\mathbf{r}_0/r^2. \tag{4b}$$

This is not the only possible separation into such factors; in the Gaussian and Lorentz-Heaviside systems, for instance, the factor $k=k'/c^2$ in Eq. (1b) is split into factors $1/c$ and $k'/c$; only the second appears in the definition of $\mathbf{B}$, and the first is put instead into the expression for $\mathbf{F}_m$ in terms of $\mathbf{B}$. Equations (3) and (4) accord with the practice in other systems and will suffice for the time being.

The magnetic measurements we are to deal with involve charges that travel in closed paths (atomic or large-scale) and constitute steady currents, as far as our apparatus can detect. Therefore only the time average counts, and Eq. (4b) may be written as

$$\mathbf{B}=k\sum i'\oint d\mathbf{l}'\times\mathbf{r}_0/r^2; \tag{5}$$

$d\mathbf{l}'$ $(=\mathbf{v}'dt)$ is a vector element of a closed path and $i'$ is the current; that is, the average charge which passes a given point per unit time. The integral is around the path, and the sum is over all such paths or *circuits*.

Besides *free* charges—able to separate entirely from their companion charges of opposite sign and thus to produce observable charge densities and currents on a macroscopic scale—we must consider *bound* charges—unable to escape entirely from their companion charges in the same atom, but able to experience displacements with respect to them on an atomic scale. Two charges of equal magnitude but opposite sign, $\pm q$, separated by a small vector distance $\mathbf{l}$, constitute an *electric doublet*. In an electric field a doublet experiences a force $\mathbf{p}\cdot\nabla\mathbf{E}$ and a couple $\mathbf{p}\times\mathbf{E}$, where $\mathbf{p}$ $(=q\mathbf{l})$ is called the *electric moment* of the doublet. A circulating (or spinning) atomic charge constitutes a small current circuit with current $i$ directed around a vector area $\mathbf{S}$. In a magnetic field a small circuit experiences a force $\mathbf{m}\cdot\nabla\mathbf{B}$ and a couple $\mathbf{m}\times\mathbf{B}$, where $\mathbf{m}$ $(=i\mathbf{S})$ is called the *magnetic moment* of the circuit.[2] In form the electric and magnetic equations are identical. Thus we might postulate *magnetic charges*, related to magnetic and electric fields as electric charges are to electric and magnetic fields, respectively; then $\mathbf{m}$ might be attributed to a *magnetic doublet* and $\mathbf{p}$ to a small *closed circuit of moving magnetic charge*. This holds also for the fields produced by doublets and circuits at large distances:

$$\begin{aligned}\mathbf{E}&=k'(-\mathbf{p}+3\mathbf{p}\cdot\mathbf{r}_0\mathbf{r}_0)/r^3,\\ \mathbf{B}&=k(-\mathbf{m}+3\mathbf{m}\cdot\mathbf{r}_0\mathbf{r}_0)/r^3.\end{aligned} \tag{6}$$

At distances of the order of magnitude of the dimensions of the doublet or circuit, the similarity breaks down.[3]

## §2. The Macroscopic Equations[4]

The fields in which we make measurements are always due to large numbers of charges, and in most cases the test object as well is a body containing many charges. In such measurements the individual charges are not observed, only their large-scale, gross effect.

Consider first the *forces on* a body in a given field, which does not include the body's own field. When the body contains an excess of free charges of one sign or carries a macroscopic current, Eqs. (3) give for the force exerted on the charges in a volume element $d\tau$

$$d\mathbf{F}_e=\rho\mathbf{E}d\tau, \quad d\mathbf{F}_m=\mathbf{j}\times\mathbf{B}d\tau, \tag{7}$$

where the *charge density* $\rho$ and *current density*

[2] The force and couple formulas are good approximations only when the dimensions of the doublet or circuit are small compared with its distance from the sources of the field.

[3] Harnwell, *Principles of electricity and electromagnetism* (McGraw-Hill, 1938), p. 350, Fig. 11.3.

[4] The transition to continuous densities is discussed in great detail by Mason and Weaver. They use only the first of the three methods to be described here; they obtain the other results by transformation of the integrals.

vector $\mathbf{j}$ are continuous functions that describe —well enough for macroscopic observations— the charge per unit volume and the magnitude and direction of the current across unit area (normal to its direction) in $d\tau$.

When the body has no net charge or current but contains polarized atoms (doublets) or atomic circuits, continuous functions $\mathbf{P}$ and $\mathbf{M}$ may be introduced to describe the electric and magnetic moments per unit volume. Three methods are available for calculating the force and torque exerted on the body by the bodies responsible for the field. In *method 1* we associate with $d\tau$ all the charges attached to atoms with centers in $d\tau$, even though some of the charges in question may have slipped across the boundary and out of $d\tau$ under the influence of the field. Since forces exerted on charges are transmitted to the atoms to which they belong, we get a force $\mathbf{P}\cdot\nabla\mathbf{E}d\tau$ and a couple $\mathbf{P}\times\mathbf{E}d\tau$ acting *on the atoms* in $d\tau$, and analogous expressions in the magnetic case. In *method 2* we associate with $d\tau$ those charges or portions of atomic circuits that happen to be in it at the moment, regardless of which atoms they belong to; the charge and current densities computed in this way are[5]

$$\rho' = -\text{div}\,\mathbf{P}, \quad \mathbf{j}' = \text{curl}\,\mathbf{M}. \tag{8}$$

(Surface discontinuities may be regarded as limiting cases of continuous, though rapid, changes, and will therefore not be introduced separately here.) Use of $\rho'$ and $\mathbf{j}'$, instead of $\rho$ and $\mathbf{j}$, in Eq. (7) gives a formula for the force on *charges in* $d\tau$, and by integration of this the total force and torque on the body may be found.

The equivalence of the formulas for the *total* force or torque obtained by methods 1 and 2 may be established by a transformation of the integrals.[6] Since method 1 gives similar expressions in the electric and magnetic cases, the equivalence also holds with magnetic and electric quantities interchanged. This justifies *method 3*, which consists in replacing atomic circuits by magnetic doublets and electric doublets by magnetic current circuits, and leads to formulas $d\mathbf{F}_e = \mathbf{j}_m\times\mathbf{E}d\tau$, $d\mathbf{F}_m = \rho_m\mathbf{B}d\tau$, where the apparent magnetic current density $\mathbf{j}_m$ and charge density $\rho_m$ are given by[7]

$$\mathbf{j}_m = \text{curl}\,\mathbf{P}, \quad \rho_m = -\text{div}\,\mathbf{M}. \tag{9}$$

The second of these expressions is more familiar but no more significant than the first. This method, like method 2, gives correctly the *total* force and torque on the test body.

Consider next the *field produced by* a charged or polarized body. At points outside the body and not too close to its surface, the atomic structure of the body is indiscernible and the continuous functions may be used in computing the fields. For free charges we have, from Eqs. (4a) and (4b) or (5),

$$\begin{aligned}\mathbf{E} &= k'\int(\rho\mathbf{r}_0/r^2)d\tau,\\ \mathbf{B} &= k\int(\mathbf{j}\times\mathbf{r}_0/r^2)d\tau.\end{aligned} \tag{10}$$

For bound charges, the three methods used before are available. If we associate with $d\tau$ the charges attached to its atoms, we get from Eq. (6)

$$\begin{aligned}\mathbf{E} &= k'\int[(-\mathbf{P}+3\mathbf{P}\cdot\mathbf{r}_0\mathbf{r}_0)/r^3]d\tau,\\ \mathbf{B} &= k\int[(-\mathbf{M}+3\mathbf{M}\cdot\mathbf{r}_0\mathbf{r}_0)/r^3]d\tau.\end{aligned} \tag{11}$$

If we associate with $d\tau$ the charges actually in it, we get

$$\mathbf{E} = k'\int(\rho'\mathbf{r}_0/r^2)d\tau, \quad \mathbf{B} = k\int(\mathbf{j}'\times\mathbf{r}_0/r^2)d\tau. \tag{12}$$

If we use the equivalent but fictitious magnetic doublets and circuits, we get

$$\mathbf{E} = k'\int(\mathbf{j}_m\times\mathbf{r}_0/r^2)d\tau, \quad \mathbf{B} = k\int(\rho_m\mathbf{r}_0/r^2)d\tau. \tag{13}$$

Equations (11)–(13) may be shown to be equivalent if the point $r=0$ is outside the region of integration.

At points inside the body it is not legitimate to neglect variations on an atomic scale, and any macroscopic field vectors can represent, at best, only space-average values. The transformation by which Eqs. (11), (12) and (13) were proved equivalent fails when $r=0$ is in the region of integration, and therefore the three must be

[5] The formula for $\rho'$ is derived by Page and Adams, *Principles of electricity* (Van Nostrand, 1931), p. 36. The formula for $\mathbf{j}'$ may be derived by calculating the current across a macroscopic surface: Only atomic circuits linked by its boundary contribute, and the result is $\oint\mathbf{M}\cdot d\mathbf{l}$; Eq. (8) follows at once. The original proof by Lorentz, *Collected works* (Martinus Nijhoff, The Hague, 1936), vol. 3, p. 117, is more complicated because a volume instead of a surface is used and because the time averaging has not been carried out in advance.

[6] In method 1 the force $\mathbf{P}\cdot\nabla\mathbf{E}d\tau$, as well as the couple, contributes to the torque about an arbitrary point. In the magnetic case the transformation holds only if div $\mathbf{B}=0$, curl $\mathbf{B}=0$; these conditions are satisfied and continue to be satisfied when $\mathbf{B}$ is replaced by $\mathbf{E}$, if the sources of the field are outside the test body.

[7] But see footnote 14.

considered separately. Since Eq. (11) was based on a formula not valid in the neighborhood of the charges, it has no physical meaning at internal points. In fact it has no mathematical meaning either, for the integral is semiconvergent.[8] Equation (12), like (10), involves no error except the substitution of continuous for discontinuous functions; these expressions may therefore be expected to represent correctly the fields with variations on an atomic scale smoothed out. Equation (13) was obtained by using the "equivalent" fictitious magnetic doublets and circuits, which are no longer equivalent to the actual electric circuits and doublets if we get too close to them, and so these expressions do not give the actual space-average **E** and **B** on the theory we have adopted. However, they do give what these space averages would be if magnetization were really due to magnetic doublets and polarization to magnetic current circuits. If we *reserve the symbols* **E** *and* **B** (at internal points) *for the sum of the right-hand members of Eqs.* (*10*) *and* (*12*), then **E** and **B** describe the actual fields at external points and the space-average values at internal points. *The sum of the right-hand members of Eqs.* (*10*) *and* (*13*) *requires different symbols*; *we shall use the symbol* **D** for the electric quantity *and the symbol* **H** for the magnetic (this is *not* the usage recommended by the A.A.P.T. Committee[9]). Outside these are identical with **E** and **B**; inside they give the space-average fields on the magnetic-doublet-and-circuit theory.[10] It may be shown that

$$\mathbf{D}=\mathbf{E}+4\pi k'\mathbf{P}, \quad \mathbf{H}=\mathbf{B}-4\pi k\mathbf{M}. \tag{14}$$

The significance of each vector as a space average may be verified by calculating the space average of the microscopic field over a "physically small" sphere. The average field of atoms and free charges outside the sphere is equal to their field at the center, which may be computed by means of the continuous densities; the result is the "Lorentz field"

$$\begin{aligned}\mathbf{E}_L&=\mathbf{E}+\tfrac{4}{3}\pi k'\mathbf{P}=\mathbf{D}-\tfrac{8}{3}\pi k'\mathbf{P},\\ \mathbf{B}_L&=\mathbf{B}-\tfrac{8}{3}\pi k\mathbf{M}=\mathbf{H}+\tfrac{4}{3}\pi k\mathbf{M}.\end{aligned} \tag{14a}$$

The contribution of atoms and free charges inside the sphere must be computed by direct averaging of the microscopic formula (4a) or (5).[11] The result just cancels the **P**- or **M**-term in one or the other of the expressions in Eqs. (14a) and leaves **E** or **H** for doublets, **D** or **B** for circuits.[12]

## §3. The Differential Equations[13]

Often it is necessary to solve simultaneously for **E** or **B** and for some of the quantities in the integrand in Eqs. (10) and (12); then the equivalent differential equations are easier to handle. In terms of **P** and **M** and of the macroscopic **E** and **B** defined by Eqs. (10) and (12), these equations are

$$\operatorname{curl}\mathbf{E}=0, \quad \operatorname{div}\mathbf{B}=0, \tag{15}$$

$$\begin{aligned}\operatorname{div}\mathbf{E}&=4\pi k'(\rho+\rho')\\ &=4\pi k'(\rho-\operatorname{div}\mathbf{P}),\\ \operatorname{curl}\mathbf{B}&=4\pi k(\mathbf{j}+\mathbf{j}')\\ &=4\pi k(\mathbf{j}+\operatorname{curl}\mathbf{M}).\end{aligned} \tag{16}$$

To solve them we must know either $\rho$ or certain data on the conductors present, either **P** or its functional relation to **E**, either **j** or data sufficient for its determination, and either **M** or its functional relation to **B**.

[8] Leathem, *Volume and surface integrals used in physics* (Cambridge, 1905), p. 17.

[9] Report of the A.A.P.T. Committee on Electric and Magnetic Units, Am. J. Phys. (Am. Phys. T.) **6**, 144 (1938).

[10] Webster, Am. J. Phys. (Am. Phys. T.) **2**, 8 (1934), finds a "difference in kind" between **E** and **D** but not between **H** and **B**. Worthing, Am. J. Phys. (Am. Phys. T.) **5**, 56 (1937), "sees no field strength . . . that can be associated with" **D**. Our derivation, on the contrary, puts **D** and **H** exactly on a par. Moreover, in a purely macroscopic theory, in which **P** and **M** are not interpreted atomically, the claim of **D** and **H** to be regarded as fundamental field vectors is as great as that of **E** and **B**: each originates from Eq. (11) by a vector transformation in the exterior region followed by extrapolation of the resulting formula to the interior. The terminology of the doublet theory, "field intensity" for **E** and **H** and "induction" for **D** and **B**, is as convenient as any for distinguishing the macroscopic vectors; all of them are "field vectors" in the sense that they define a vector field.

[11] S. L. Quimby, *Lectures on electromagnetic theory* (privately printed, Columbia Univ., 1933), pp. 33–34, 114.

[12] The physical importance of space-average values should not be overestimated. The region averaged over includes the positions of the charges $q'$, where classical formulas, such as Eq. (1) and the microscopic equations of Lorentz, are of doubtful validity; cf. Mason and Weaver, p. 5, and O'Rahilly, *Electromagnetics* (Longmans, 1938), pp. 451–481. For this reason the advance involved in substituting **B** for **H** as fundamental vector may be rather trivial.

[13] These may be derived either from the integral equations (see any textbook on vector analysis) or from the flux theorems and circuital relations satisfied by the microscopic vectors (Page and Adams, pp. 18–21, 41–42, 138–139, 160–161, 258–259).

The equations may also be written in terms of $\mathbf{P}$ and $\mathbf{M}$ and of the vectors $\mathbf{D}$ and $\mathbf{H}$ defined by Eqs. (10) and (13):[14]

$$\begin{aligned} \operatorname{curl}\mathbf{D} &= 4\pi k'\mathbf{j}_m & \operatorname{div}\mathbf{H} &= 4\pi k\rho_m \\ &= 4\pi k' \operatorname{curl}\mathbf{P}, & &= -4\pi k \operatorname{div}\mathbf{M}, \end{aligned} \tag{17}$$

$$\operatorname{div}\mathbf{D} = 4\pi k'\rho, \qquad \operatorname{curl}\mathbf{H} = 4\pi k\mathbf{j}. \tag{18}$$

Equation (14) enables us to restate the functional relations $\mathbf{P}(\mathbf{E})$ and $\mathbf{M}(\mathbf{B})$ in the form $\mathbf{P}(\mathbf{D})$ and $\mathbf{M}(\mathbf{H})$ and to find $\mathbf{E}$ and $\mathbf{B}$ when the equations have been solved. Although Eq. (18) relates $\mathbf{D}$ and $\mathbf{H}$ to the *free* charge and current densities in the same way that Eq. (16) relates $\mathbf{E}$ and $\mathbf{B}$ to the *total* densities $(\rho+\rho', \mathbf{j}+\mathbf{j}')$, it is not correct to identify $\mathbf{D}$ and $\mathbf{H}$ with the contribution of the free charges to $\mathbf{E}$ and $\mathbf{B}$, except in special cases in which the right-hand members of Eq. (17) vanish everywhere (boundary layers included). Some definitions of $\mathbf{D}$ and $\mathbf{H}$ given in recent textbooks and discussions are objectionable for this reason.

A third form of the equations, simpler mathematically and therefore more useful for the practical solution of problems, may be obtained by expressing them in terms of $\mathbf{E}$ and $\mathbf{D}$, $\mathbf{B}$ and $\mathbf{H}$. The result is Eqs. (15) and (18) together with a functional relation $\mathbf{D}(\mathbf{E})$, $\mathbf{H}(\mathbf{B})$. When the equations have been solved, Eq. (14) gives $\mathbf{P}$ or $\mathbf{M}$. In a region where there are no free charges, $\mathbf{E}$ and $\mathbf{H}$ are irrotational (no curl) and $\mathbf{D}$ and $\mathbf{B}$ are solenoidal (no divergence); therefore the electrostatic and magnetostatic problems may be made formally identical by putting the magnetic relation into the form $\mathbf{B}(\mathbf{H})$. This amounts to taking the doublet field as the independent variable in each case. Such mathematical juggling has to do only with the development of a technic for solving problems—a stage of the theory in which the atomic interpretation of the various vectors is of no concern to us.

The irrotational and solenoidal properties of the vectors have been made the basis of a number of geometrical representations of the fields and of picturesque descriptions borrowed from analogous problems in hydrodynamics. These devices are helpful if they are used properly, dangerous if they are taken too seriously. They have no bearing on the atomic interpretation or the "dimensions" of the vectors.

The location of $4\pi$ and of the $k$'s in the equations developed so far may be remembered by the following rule. Each electric or magnetic symbol represents either a *source* quantity ($\rho$, $\mathbf{j}$, $\mathbf{P}$, $\mathbf{M}$), which describes the charge or current distribution at a point, or a *field* quantity ($\mathbf{E}$, $\mathbf{B}$, $\mathbf{D}$, $\mathbf{H}$), which describes the electric or magnetic situation produced at a point by charges located largely at other points. The constant $k'$ or $k$ accompanies any operation that converts a source quantity to a field quantity. When the operation is the evaluation of a field quantity by summation or integration over the sources, $k'$ or $k$ occurs alone. When the operation is the evaluation of the curl or divergence of a field quantity (or its equivalent a line or surface integral, as in the Gauss theorem) in terms of a source quantity, a $4\pi$ occurs also.

If we prefer to get rid of the $4\pi$'s and $k$'s in the differential equations, we may go back to Eqs. (3) and (4) and divide the $k$'s differently by putting a $4\pi k'$ or $4\pi k$ in Eq. (3) and a $1/4\pi$ in Eq. (4). The new field vectors $\mathbf{e}$, $\mathbf{b}$, $\mathbf{d}$ and $\mathbf{h}$ will be the old ones divided by $4\pi k'$ or $4\pi k$, and we have, instead of Eqs. (14), (15) and (18),

$$\mathbf{d} = \mathbf{e}+\mathbf{P}, \qquad \mathbf{h} = \mathbf{b}-\mathbf{M}, \tag{19}$$

$$\operatorname{curl}\mathbf{e} = 0, \qquad \operatorname{div}\mathbf{b} = 0, \tag{20}$$

$$\operatorname{div}\mathbf{d} = \rho, \qquad \operatorname{curl}\mathbf{h} = \mathbf{j}. \tag{21}$$

The $4\pi$'s and $k$'s also vanish from Eqs. (16) and (17); a factor $1/4\pi$ replaces the $k'$ or $k$ of Eqs. (4)–(6) and (10)–(13).

There is no point in choosing a combination of vectors, such as $\mathbf{E}$, $\mathbf{d}$, $\mathbf{B}$, $\mathbf{h}$, that puts the constants partly in one set of equations and partly in another. Unfortunately, such a procedure has been recommended by the A.A.P.T. Committee and has found its way into textbooks. With this mixed set of vectors there is no longer a simple rule for remembering the location of the constants, and the discussion of §2 becomes entangled in irrelevant and meaningless symbolism.

[14] The first of these in its most general form, $\operatorname{curl}\mathbf{D} = 4\pi k' \operatorname{curl}\mathbf{P} - \dot{\mathbf{H}} - 4\pi k\dot{\mathbf{M}}$, shows that on the magnetic charge theory the magnetic current density is really not $\operatorname{curl}\mathbf{P}$ but $-(k'/k)\operatorname{curl}\mathbf{P}$; for $\dot{\mathbf{M}}$ is obviously a magnetic current density produced by motion of the doublet charges. The symmetry between electric and magnetic quantities is improved by using the Gaussian definitions of the magnetic vectors.

## §4. Special Materials

To solve a problem on dielectrics or magnetic materials, we must know the functional relation $\mathbf{D}(\mathbf{E})$ or $\mathbf{B}(\mathbf{H})$. In most nonferromagnetic materials the assumption that this relation is linear is a good approximation, supported both by experiment and by atomic theory.[15] The further assumption usually made, that the material is isotropic, is a poor approximation for some solids used in practice and is inapplicable to crystals. For linear isotropic materials $\mathbf{D}=\epsilon\mathbf{E}$, $\mathbf{B}=\mu\mathbf{H}$, where $\epsilon$ and $\mu$ are independent of the fields. Being dimensionless, $\epsilon$ and $\mu$ are also independent of the units, and they may be defined equally well by $\mathbf{d}=\epsilon\mathbf{e}$, $\mathbf{b}=\mu\mathbf{h}$. The differential equations for linear isotropic materials are

$$\text{curl}\,\mathbf{E}=0, \qquad \text{div}\,(\mu\mathbf{H})=0, \tag{22}$$

$$\text{div}\,(\epsilon\mathbf{E})=4\pi k'\rho, \qquad \text{curl}\,\mathbf{H}=4\pi k\mathbf{j}. \tag{23}$$

The removal of $\epsilon$ and $\mu$ outside the operators div and curl is permissible only in a region where they are independent of position, that is, in the *interior* of a homogeneous body at a uniform temperature. In the inhomogeneous transition layer at the surface, the equations just given lead to the familiar "boundary conditions" as limiting cases.

In a homogeneous linear isotropic medium, Eqs. (22)–(23) expressed in terms of $\mathbf{E}$ and $\mathbf{B}$ are the same as without the medium except that $\rho$ is replaced by $\rho/\epsilon$ and $\mathbf{j}$ by $\mu\mathbf{j}$. It does not follow in general that the resulting $\mathbf{E}$ or $\mathbf{B}$ is $1/\epsilon$ or $\mu$ times the value without the medium. It does follow if a single homogeneous linear isotropic medium fills all portions of space where there is a field. This condition is satisfied by conductors immersed in a dielectric, but not by nonconducting charged or polarized bodies immersed in one (since the dielectric is excluded from the interiors of the bodies). It is also satisfied by a ring specimen of magnetic material surrounded by a toroidal winding. In most cases the solution of the problem is much less simple. The commonly encountered statement of the Coulomb law for charged bodies immersed in a medium, $\mathbf{F}_e=k'qq'\mathbf{r}_0/\epsilon r^2$, with $\epsilon$ in the denominator, is correct only as an approximation when the dimensions of the bodies are small compared with the distances between them and when the medium has the very special properties already enumerated. The usual statement of it appears to imply the following conclusions, all false: (1) that it has general enough validity to serve as a basis for a complete macroscopic theory of electrostatics; (2) that it is theoretically exact in the sense in which Eq. (1a) is exact; (3) that it describes some mysterious ability of the medium to alter the force exerted by one charge on another; (4) that if it is summed over all the charges $q$ of one body *of finite volume* immersed in a dielectric, and over all the charges $q'$ of another such body, the result is a correct expression for the total force exerted on one of the bodies in the presence of the other and of the dielectric. Actually, (1) this equation is useless for anisotropic, permanently polarized or piezoelectric substances, and for systems in which different dielectrics fill different regions or in which a dielectric is subject to inhomogeneities, including temperature gradients; (2) theoretically (and the theory is confirmed by experiment), the linear relation between polarization and field—on which the equation in question depends for its validity—is only an approximation and ceases to hold at high fields; (3) the equation describes not the force exerted on $q$ *by the charge* $q'$—this force is always given by Eq. (1a), whether or not a medium is present—but the resultant force exerted by $q'$ *and the polarized molecules of the dielectric*, together; (4) in the special cases in which this *additive* effect of the polarized molecules is equivalent to *multiplication* of the original force by $1/\epsilon$, the equivalence depends on the fact that the dielectric may, without appreciable error, be supposed to fill all space; for nonconducting bodies whose volumes are an appreciable fraction of the cubes of their mutual distances, this supposition is no longer justified, and the effect of introducing a dielectric is more complicated than a mere decrease of every force in the ratio $1/\epsilon$.

At the other extreme is the temptation to apply the Coulomb law with a factor $1/\epsilon$ to isolated ions surrounded by polarizable molecules. Obviously, the macroscopic theory affords no justification for such a procedure; the "point charges" for which the formula is valid must, indeed, be small compared with their distances of separa-

[15] Van Vleck, *Electric and magnetic susceptibilities* (Oxford, 1932).

tion, but they must still be large enough to justify the treatment of the dielectric as a continuum. The ion problem must, therefore, be attacked by returning to Eq. (1a) and taking account of the atomic structure of the medium. If, in special cases, a formula like that of the macroscopic theory is obtained, it is a new result, which could not have been derived from (though it may have been suggested by) the macroscopic theory.

The quantities $4\pi k'/\epsilon$ and $4\pi k\mu$ occur in problems on homogeneous linear isotropic materials. It may be convenient in such problems to represent them by single symbols, to regard them as constants of the material, and to regard $4\pi k'$ and $4\pi k$ as the special values assumed by them in vacuum, which thus becomes a special "material." This figure of speech has its place in the treatment of such special problems. It has no place in the basic equations of the theory: here $k'$ and $k$ are simply proportionality constants in the fundamental laws of force. If the vectors **e**, **d**, **b**, **h** are used, the combinations $4\pi k'/\epsilon$ and $4\pi k\mu$ do not occur often enough to warrant the introduction of special symbols for them.

## §5. Units and Systems of Units

We must discuss separately *units of charge* and *complete systems* of units. The common *units of charge* are the *Gaussian* (G) (also called electostatic), the *Lorentz-Heaviside* (LH), the *electromagnetic* (em) and the *practical* unit or coulomb. The first three are so chosen as to make $k'=1$, $k'=1/4\pi$, and $k=1$, respectively, when the mechanical units are cgs. The coulomb is 1/10 of an em unit. It follows that when the coulomb is used with mks mechanical units, $k=10^{-7}$ newton/amp$^2$ and $k'=kc^2\doteq 10^{-7}\times(3\times10^8)^2=9\times10^9$ newton meter$^2$/coul$^2$.

The only *complete systems* of units and symbols now in common use are the G, LH, and mks-coulomb (mksc) systems. Equations (1)–(18) and (22)–(23) are correct in the mksc system with the proviso that the symbols **D**, **H**, **m** and **M** must have the meanings assigned to them here. The G and LH systems divide the constants differently in the magnetic equations, in such a way as to give each magnetic quantity the same dimensions as the corresponding electric quantity. To obtain the general equations for systems of this sort, we replace $k$ by $k'$ and write $1/c$ in front of each symbol that represents a *velocity*, a *current* or a *current density*, or (in the complete field equations) a *time differentiation:* thus Eq. (16) becomes curl $\mathbf{B}=4\pi k'(\mathbf{j}/c+\text{curl }\mathbf{M})$.[16] Then $k'=1$ gives the G system and $k'=1/4\pi$, the LH system.

The original equations with $k'=1$ give the *electrostatic* system, with $k=1$, the *electromagnetic* system, except that some magnetic quantities in the former and electrostatic quantities in the latter differ from ours by constant factors. The electrostatic system is little used outside of electrostatics, where it is identical with the Gaussian. The electromagnetic system is used chiefly in circuit theory to avoid the Gaussian $c$'s; the equations needed for this purpose may be obtained from the Gaussian by the substitutions $q=cq_{\text{em}}$, $i=ci_{\text{em}}$, $\mathbf{j}=c\mathbf{j}_{\text{em}}$, $\mathbf{E}=\mathbf{E}_{\text{em}}/c$. For practical purposes, these two *systems*, as such, may be ignored altogether, and the few electromagnetic *units* that are needed and that differ from the Gaussian may be introduced in the manner just indicated. The usual procedure—introduction of the G system as a "combination" of the electrostatic and electromagnetic—is indirect and misleading.

The mksc equations may be expressed in terms of **d** and **h** (the "**D**" and "**H**" of the A.A.P.T. Committee report) by the substitutions $\mathbf{D}=4\pi k'\mathbf{d}$, $\mathbf{H}=4\pi k\mathbf{h}$; and in terms of vectors $\mathbf{d}'=\mathbf{E}/k'+4\pi\mathbf{P}$, $\mathbf{h}'=\mathbf{B}/k-4\pi\mathbf{M}$ (the "**D**" and "**H**" of some writers) by the substitutions $\mathbf{D}=k'\mathbf{d}'$, $\mathbf{H}=k\mathbf{h}'$. Those who insist on putting the "material" properties of empty space into the fundamental equations may make the further substitutions $k'=1/4\pi\epsilon_0$, $k=\mu_0/4\pi$ when **d** and **h** are used, or $k'=1/\epsilon_0$, $k=\mu_0$ when **d**′ and **h**′ are used; then $\epsilon$ and $\mu$ are to be written $\epsilon_r$ (or $\kappa$) and $\mu_r$, or $\epsilon/\epsilon_0$ and $\mu/\mu_0$, and **M** is to be replaced by $\mathbf{M}/\mu_0$.[17] The

[16] Setting $k=k'$ gives the equations of a system in which the units of charge, force and distance are the desired ones, but the unit of time is the time it takes light to travel unit distance; the factors $1/c$ convert the ordinary velocities, currents, etc., to this new time scale. The system used by Mason and Weaver differs from the LH system in that the $1/c$ is not written in front of current and current density but is put, instead, into the formulas for these in terms of charge and time.

[17] If two long, thin magnets with uniform magnetic moments per unit length, $m_0$ and $m_0'$, are separated by distances large compared with their thicknesses, the equivalent magnetic charge distribution given by Eq. (9) reduces to magnetic point charges $\pm m_0$, $\pm m_0'$ at the ends or *poles* of the magnets. These fictitious charges may be used to compute the forces exerted by the magnets on each other; and if the magnets are immersed in an infinite homogeneous linear isotropic magnetic medium, the atomic circuits in the medium may be replaced by the equivalent doublets, since points inside the magnets are "outside" the medium in the sense of §2. By analogy with electrostatics, therefore, the repulsive force apparently exerted by the positive poles on each other is $km_0m_0'/\mu r^2$. If the reciprocal of the coefficient of $m_0m_0'/4\pi r^2$ or of $m_0m_0'/r^2$ is defined as the "absolute permeability" of the medium, we see that the absolute permeability is $\mu/4\pi k$ in the one case and $\mu/k$ in the other. This differs from the "absolute permeability" for current circuits of negligible thickness immersed in a medium. which is defined on the

writer can see no reason for using these symbols if the object is to give a faithful description of phenomena rather than to force the phenomena into a traditional framework of jargon and symbolism.

"**H**" in the electrostatic system and "**D**" in the electromagnetic vary somewhat with the author but are likely to be **h**′ and **d**′ or **d**. The last is based on Maxwell's mechanical theory of electricity in which a polarization **P** was attributed to the dielectric and an additional polarization $\mathbf{E}/4\pi k'$ to the "ether"; the total, **d**, was the "electric displacement." An obsolete theory is scarcely an adequate basis for definitions when we are constructing a *new* system of units and equations.

In the LH system **D** and **d** are identical, and "rationalization" is accomplished without having to put a $4\pi$ into Eq. (3). It is in an attempt to generalize the LH rationalization that the mixed vector system **E**, **d**, **B**, **h** has been introduced. But the logical generalization is the system **e**, **d**, **b**, **h**, which in mksc units means abandonment of the volt, ohm, farad and henry. If this is too radical a step to take, then we may as well admit that we are unwilling to pay the price of rationalization. The number $4\pi$ does not cease to be $4\pi$ just because somebody decides to call it $10^7\mu_0$.

Dimensions require little comment. If force, length, time and charge [F, L, T, Q] are taken as fundamental, the electrostatic Eq. (16) may be checked dimensionally as follows. On the left, div is a differentiation with respect to distance, dimensions [1/L]; **E** by its definition is [F/Q]; resulting dimensions, [F/LQ]. On the right, $k'$ by Eq. (1a) has dimensions $[FL^2/Q^2]$, $\rho$ has dimensions $[Q/L^3]$; product, [F/LQ], which checks. Electric moment is [QL], and **P** is therefore $[QL/L^3]=[Q/L^2]$, so that div **P** is $[Q/L^3]$, the same as $\rho$. This method of checking is useful in the general equations and in the mksc system. When $k$ or $k'$ is assigned a value 1 or $1/4\pi$, whose presence or absence in an equation is either of no consequence or not capable of verification by the dimensional method, it may conveniently be made dimensionless. Charge thereupon receives mechanical dimensions, different in the electrostatic and in the electromagnetic systems, so that conversions from one system to the other may also be checked to some extent by the dimensional method.

## §6. Conclusion

The writer is not arguing for or against the use of mks units, for or against rationalization. He believes that whatever units are used, the equations should be developed in a general enough form to be valid (with as few changes as possible) in other units as well; and that whether the $4\pi$'s are put in the integral equations or in the differential, they should all be put in the one or all in the other—and a person should know where they are. His rejection of the **D**, **H**, $\epsilon_0$ and $\mu_0$ of the A.A.P.T. Committee report has been based on these principles.

If the theory is developed in the manner outlined here, calculations in the mksc system can be carried out by direct use of the general equations, by merely inserting the numerical values of $k$ and $k'$. At the same time, the special forms of the equations corresponding to other systems can be obtained easily by the rules given in §5. The rule for location of the $k$'s and $4\pi$'s is simple, and the only numerical constants that must be remembered in the mksc system are the speed of light (which must be remembered in any system) and $10^{-7}$; the relation $k'=kc^2$ is easy to remember, for $k'$ in the equation for electrostatic force takes the place of $k$ and two velocities in the magnetic equation. Whether the unrationalized system of vectors **E**, **D**, **B**, **H** or the rationalized system **e**, **d**, **b**, **h** is used, $k'$ and $k$ are the most convenient constants. The splitting of the rational fraction $10^{-7}$ into two factors, both irrational, is no aid to the memory and is a curious interpretation indeed of the word "rationalization."

The writer wishes to express his appreciation to Professor C. W. Ufford of Allegheny College for a number of stimulating discussions and for helpful criticism of an earlier version of this article.

basis of Eqs. (22)–(23) as $4\pi k\mu$ or $k\mu$. In order to bring the two "absolute" permeabilities into agreement when $k$ is neither $1/4\pi$ nor 1, it is necessary to define magnetic moment not as $i\mathbf{S}$ but as $4\pi ki\mathbf{S}$ ("rationalized" mks system) or $ki\mathbf{S}$ (electrostatic and "unrationalized" mks systems). The writer contends that the absolute permeabilities have no physical significance and, therefore, that there is no reason for bothering to bring them into agreement or even to define them; that, in fact, it is better not to bother, because to do so implies the physical importance of concepts whose actual physical importance is nil. Incidentally, the definition of a pole as an *end* and of pole strength as *magnetic moment per unit length* makes it unnecessary to conjure the pole into existence by dissection of a fictitious doublet. *Poles* as thus defined exist in nature; *magnetic charges* are fictitious. The law of force for magnets immersed in a medium can be derived equally well by using the equivalent circuits; the apparent difference in behavior between a magnet and a coil is due entirely to the fact that the magnet excludes the medium from its interior, where the magnetic induction is very large.

# Teaching Engineering Physics

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SOME time ago I read a description of a projected super-automobile highway to be built across the continent. Its many features included an arrangement for cars to get on the highway at intervals of ten miles, and if one coming off had gone past the station for which toll had been paid, extra fare was collected. Once on the highway, the driver selected the lane appropriate to the speed at which he wished to drive. As the car sped along, stationary road lights operating automatically illuminated the road a distance ahead that was proportional to the car's speed. If another car approached too close behind, red highway lights gave the proper warning, etc., etc. This was, indeed, the super-highway. In reading the article, I noted two outstanding characteristics. First, the designer claimed that he had not incorporated any device or construction plans that had not already been used and tested on other highways. Second, the road had not been built; it was only a projected super-highway.

In this discussion of the teaching of engineering physics, it is intended that the design shall comply with the first of the preceding characteristics. Also, so far as is known, no course like it has been built.

It is assumed that every teacher has objectives. Sometimes they have been carefully and explicitly formulated. More often they are implicit and hazy; the teacher merely teaches a course. There follows four objectives which, in my judgment, engineering physics should strive to reach.

*Objective 1.—A thorough mastery of the basis of general physics.* This is a generalized and fairly safe statement until one attempts to explain what it means. By it, I mean three things: (*a*) an understanding of the abstract principles; (*b*) recognition and ability to use these principles in theoretical problem situations; (*c*) ability to identify these textbook principles with the ones actually operating in engineering situations and to use them in these actual situations.

*Objective 2.—The student should learn a great deal about the measuring instruments of physics that are useful to the engineer, and should develop some skill in handling them.*

*Objective 3.—The student should develop a scientific attitude.* This is another safe generalization that practically every science teacher includes in his objectives. However, I shall leave this safe ground and attempt to specify what a person having a scientific attitude will do. One who has a scientific attitude will know the nature of evidence and the necessary and sufficient conditions for establishing a fact. Concerning an allegation drawn from data, he will be able to judge whether the evidence is sufficient to make the statement true, probably true, probably false or certainly false, or whether the evidence is insufficient to make a decision. In brief, one having a scientific attitude knows the necessary and sufficient conditions for proof.

The individual with a scientific attitude will seek new data on old, unsolved problems and will treat these and all other data objectively. He will see and recognize scientific principles operating in everyday life. He will show an active interest in learning about new things—how they operate and where the "mainspring" is—and will evidence this interest in his conversation and his reading. He will also devise methods for testing his theories. This is what I mean by a scientific attitude.

*Objective 4.—The course should kindle the imagination and fire the creative abilities of the students.*

## Teaching Tools and Assets

Let us inventory some of the tools and assets that the teacher brings to the task of teaching engineering physics.

*1.* It is now possible to make a fairly accurate prediction of what the sophomore engineer will achieve in terms of grades in engineering physics before he enters the first physics recitation; that is, the teacher can, if he wishes, make a fairly accurate estimate of the quality of the raw material before him. The knowledge that students are prospective A's, B's, C's or D's should be

useful because instructional methods can and should be modified to suit the students' individual needs.

For some years many studies have been conducted in attempts to determine what characteristics make for success in college physics. The best judgments of objective teachers indicate that some of the characteristics are: ability to handle symbols in mathematical equations, general interest in physical science, ability to employ abstract logic, ability to comprehend reading material (so-called reading comprehension) and ability to visualize mechanical and geometric forms.

It was upon four of these characteristics that the first *Iowa Physics Aptitude Test* (1926) was based. Part One of the test dealt with mathematical preparation; Part Two, with reading comprehension of physics materials; Part Three, with ability to employ abstract logic; and Part Four, with interest in physical science. Repeated analyses of the results of these tests force us to conclude that Part One, mathematics training, and Part Four, interest in physical science, are the only parts contributing significantly to the ability of the whole test to predict success in physics. These two parts taken together, have a high predictive ability. A mathematics test has also been devised that inventories the specific abilities used in college physics. When the physics aptitude test is taken together with the mathematics inventory test, highly accurate predictive results have been found.[1]

L. L. Thurstone, the psychologist, has classified and provided ways to measure the alleged human intellectual faculties as follows: form perception, or the ability to identify identical forms in line drawings; the number factor, measured by the ability to divide, multiply and add; the verbal factor, measured by one's vocabulary; the space factor, measured by the recognition of forms in line drawings that have been rotated and translated out of their original patterns; the memory factor, measured by how well one associates names and numbers; the inductive factor, measured by ability to discover the common quality in a figured pattern made from associated line dashes; and the reasoning factor, measured by the ability to predict possible motions of mechanical devices. Thurstone maintains that these are the primary mental abilities.

The space factor comes closest to measuring what some investigators have called visual imagery. Teachers of physical science and engineering seem agreed that the ability to build and hold images of geometric figures is a factor of primary importance; but, so far as is known to the writer, all attempts made to prove that this factor makes for success in physics have been conspicuously unsuccessful. In recent measurements made on 170 freshmen engineering students in Pennsylvania State College, Thurstone's space factor showed a correlation with grades in freshman engineering subjects of only 0.23. However, it was determined that the spatial relations factor was more highly developed in these students than in the average high school senior.

Last year our Department of Physics conducted a rather elaborate testing program using the Minnesota paper form board test in an effort to determine the usefulness of a visual imagery test for predicting success in college physics. The correlation coefficient between the tests and the semester's physics grades was 0.26. At least three other studies have been made in the University of Iowa in an attempt to determine what part this factor played in success in college physics. The studies were conspicuous for their negative results. The writer is quite baffled by these findings, as it certainly seems reasonable to believe that the ability to build and hold geometric images while one works with them is a strong factor in the mental equipment of successful scientists and engineers.

The writer also finds that the scores on the general tests for reading comprehension given to freshmen entering the University of Iowa appear to have no significant correlation with academic success in college physics. However, these reading scores correlate very highly with academic success in subjects such as history and sociology.

The performance of the last 200 engineering students who took physics at the University of Iowa and whose freshman mathematics and engineering drawing records were available have been analyzed with the following results. The correlation coefficient for the mathematics and

[1] C. J. Lapp, J. Eng. Ed. **28**, 222 (1937).

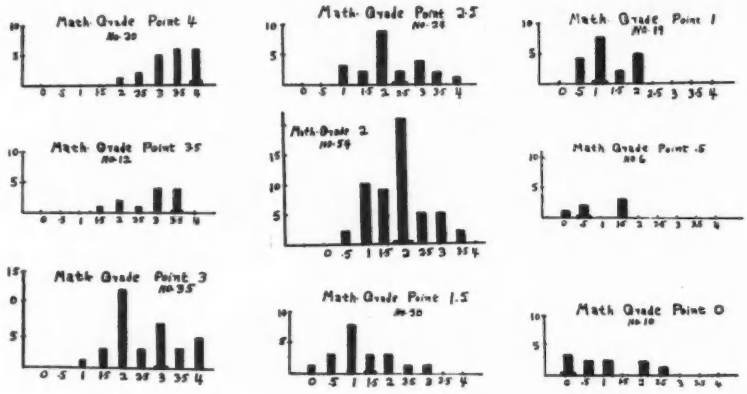


FIG. 1. Distribution of physics grades for the group making a particular mathematics grade.

physics grades is 0.68. The correlation coefficient for the physics grades and the mathematics and engineering drawing grades taken together is 0.70; that is, the predictive value of the mathematics grade is only slightly enhanced by the inclusion of the drawing grade. In Fig. 1 each bar chart shows the distribution of physics grades for the group making a particular mathematics grade. In these charts grades A, B, C and D are represented by the numerals 4, 3, 2 and 1, respectively. The abscissas represent physics grades, while the ordinates represent the number of students involved. Thus, the diagram in the upper left-hand corner indicates that the 20 students who made A in mathematics had physics grades distributed as follows: 6 made A, both semesters, 6 made A one semester and B the other, 5 made B, etc.

It appears from these results that, if the physics teacher wishes to section engineering students on the basis of their probable performance, it can be done with a high degree of reliability on the basis of their mathematics grades.

*2. Engineering students generally are able tudents.*

This has been shown repeatedly by comparing the freshman entrance-test scores of engineering and other students. For eight years the Cooperative Test Service has distributed carefully prepared examinations in college physics that have been taken by tens of thousands of students in at least 500 different institutions. The achievement has been analyzed by groups, and in every case the achievement of the engineering students has stood substantially above that of any other group on all of the divisions of physics except modern physics. This one exception is doubtless due to lack of instruction in that subject matter. These results would probably be even more conclusive if the tests had been difficult enough to give the engineering students opportunity to demonstrate their full superiority. Experienced teachers agree that it is much more stimulating to teach able students than dull ones. Also the able student requires less of the "pick-and-shovel" type of work, hence the teacher has more time available for the use of teaching methods designed to achieve Objectives *3* and *4*. Therefore, the high native intelligence should be treated as a distinct asset.

*3. Never in the history of physics teaching has there been so much good lecture demonstration material available as at the present time.*

Articles on modern and successful lecture demonstrations are continually appearing in the *American Journal of Physics*. The sponsorship by the American Association of Physics Teachers of the book, *Demonstration Experiments in Physics* (McGraw-Hill) has added very substantially to the literature of the field. Demonstration apparatus and instruments are readily available. This part of physics teaching is abundantly supplied with such materials.

*4. A larger variety of problem materials is now available than ever before.*

New books are continually being published; and although a great deal of repetition occurs, each new book is contributing in varying degrees to the worthwhile problem material.

*5. In recent years the motion picture has become an educational tool which cannot be denied its rightful place in teaching.*

General Electric, Westinghouse, Eastman Kodak, United States Steel, Erpi Sound Films, Inc. and others have produced a large number of films of varying value, many of which are exceedingly useful in teaching.

*6. There is coming into existence a great deal of reading material which has high value.*

Examples are McGee's *Source Book in Physics*, the many volumes on industrial applications and on modern physics, and the survey articles of elementary and intermediate grades now appearing in various scientific journals. Each year the lay periodicals carry increasing references to physics, stressing the increasing importance of its applications. These available library readings are assets which I fear have been overlooked by too many teachers.

It is not pretended that these are the only teaching tools and assets of the physics teacher. The ones listed, however, are of primary importance; and the present-day teacher of engineering physics will do well to consider seriously how the items listed can be used to improve and modernize his methods of instruction.

## Boundary Conditions

The term *boundary conditions* is not borrowed from the vocabulary of the professional educator but here has the same meaning as when used in connection with a problem situation in physics. It means the limiting conditions that must be recognized and satisfied by a satisfactory solution —the conditions under which the teacher of engineering physics must, of necessity, operate. Some of the important ones will be listed.

*1.* The students are usually sophomores who are taking calculus concurrently. They have a heavy schedule that makes severe demands on their time. Some of their classes demand material to be handed in, much of which is easier and more fun to prepare than the physics assignments. The physics professor has a lien on about 15 hours per week of the student's time and, if he is to get it, must be both wise and persistent. Carefully considered lesson assignments must be made in terms of the difficulty of the material to be covered and a check-up system of some sort enforced to guarantee daily preparation. This system must begin functioning the first day and continue reasonably but relentlessly. There will be plenty of competition for the student's time.

*2.* The sophomore engineer is trade conscious; he lustily sings "St. Patrick was an Engineer" and wants it known that he belongs to the crowd that "screwed the lawyers to the bench." The immediate desire to be taught things that are obviously utilitarian is so strong in his mind that he frequently becomes vocal; but, vocal or repressed, the attitude is present and with it a reckoning must be made. The wise teacher will attempt to satisfy, rather than repress, this attitude.

*3.*[2] There are certain elements in one of the theories of learning that can be treated as boundary conditions. Industrial processes proceed at an optimum rate when regulated and controlled by one who knows their bases. Similarly, other things being equal, student learning in a classroom proceeds faster when directed by one who understands the laws of learning. It would seem that there are at least three places in this area where teaching breaks down.

(*a*) *Learning is connecting.*—There is a stimulus and a response; the connection between the two is called a *bond*. A child sees the letter *A* and the teacher urges the child to make the sound *A*. The stimulus is a blotch of black on a piece of white paper and the response is the sound *A*. In a similar way, there is a different stimulus and a different response for other blotches of black. Three particular blotches are taken together and the response is the word *boy*. That is just the point. One stimulus has been made from three. Small bonds are continually and quite unconsciously being made into large ones. The minds and nervous systems of violin players, expert stenographers and teachers are well stocked with big bonds made from hundreds of smaller ones.

Learning ordinarily proceeds by small, definite, logical steps. Many small bonds are made which attach themselves to others to form a chain. If the thing learned becomes commonplace with usage, all of the links may be fused into a single big link or bond. If the possessor of the big bond teaches this particular area of learning he may inadvertently omit some of the steps necessary to a new learner and at that point he and the student will not understand each other. The teacher of physics states that the "conditions for equilibrium are that the vector sums of the forces and of the torques acting on a body are zero." To the beginner, this statement is quite meaningless, and several hours of careful study are usually

[2] The correctness of the statements in this section has been checked by Professor J. B. Stroud, Department of Educational Psychology, State University of Iowa.

required before the conditions for equilibrium acquire meaning. This omission of necessary steps in instruction is one of the frequent causes of misunderstanding between teacher and student. If space permitted, many interesting illustrations of this point could be made.

(*b*) *Identifiability.*—The first thing a nervous system does with a stimulus is to try to identify it. There may be a strong stimulus-response bond in existence but it cannot act until the identity of the stimulus is established. A small animal approaching us in the twilight may be a pet kitten, a neighbor's puppy or a polecat. The response that comes will depend on the identification of the stimulus. Consider my friend's three-year-old son who was much interested in airplanes. One evening he excitedly pointed his chubby finger to the sky and exclaimed "Airplane, airplane, airplane—ah, birdie!" In the middle of his sentence he had discovered a mistaken identity. Identifiability comes strongly into all learning. Words do not always mean the same to different people, and even words with well-established meanings may be compounded into meaningless sentences. The phrase "time-rate of change of displacement" is not usually identified as velocity until the latter concept is well established.

One of my teachers once tried to describe the forces involved in rowing a boat. As a language medium he used English and his hands. He appeared to apply forces to the oars and the boat, the boat applied forces to him and the oars, the water applied forces to the oars and the boat—and the oars applied forces to everything. It sounded like one verse of "Old Macdonald Had a Farm,"

> with a force, force here and a force, force there;
> here a force, there a force, everywhere a force,
> force, . . .

If, for the motions of his hands, the teacher had substituted a good line drawing done with chalk on the blackboard, identification would have been easy. As it was, so far as I know, nobody ever discovered what he was talking about.

A learning bond cannot be established between two points in a vacuum. A student must have a *good square look* at the thing to be identified, cut away from its surroundings, before he can be expected to identify it in another situation with new surroundings. A teacher bumps together ivory balls of two different sizes and says, "Behold, the conservation of momentum!" The student may not be sure whether it is the whiteness of the balls, the sound of the impact, the transfer of energy or something else that is to be identified with conservation of momentum. Identifiability increases with repetition and with practice in identifying the things in question in new situations. Things may also be made more identifiable by close analysis where hidden features are brought into light.

(*c*) *Readiness.*—If two stimulus bonds are of equal strength and both are stimulated, the one that is the *more ready to act* will give the stronger response. The response of eating comes easily if one is hungry and with difficulty if one is not. Learning is like that. The same student varies from time to time in his readiness to form new bonds, that is, to learn. Any emotion—fear, embarrassment, anger, resentment—may bring about a state of unreadiness as does also fatigue, excitement, the anticipation of a coming vacation or an athletic contest. Many of these things will be recognized at once by the skilful teacher. But some of them, particularly some of the emotional situations, are difficult to analyze, appear at unexpected times and form effective blocks to learning in a classroom. If learning is to take place easily, there must be no emotional blocks between the student and the teacher. If the student feels that he has been treated unfairly, has not been given a chance to explain, has been graded too severely, has been embarrassed or caused to lose face before his associates, or if he thinks the teacher is an intellectual high-brow strutting before the class to display his learning, his superior intelligence, his fine voice or curly hair, the student cannot help forming an emotional block which is surprisingly effective in keeping him from being ready to learn. There must be a mutual feeling of respect and good will. The teacher must be respected in the intellectual, social and moral areas, to the point that if war were suddenly declared and the teacher were to become a commissioned officer in a company the privates of which were the members of his class, he would be perfectly safe with them at the front line trenches. Some of you know exactly what I mean.

(*d*) *Human forgetting.*[3]—The fourth and last

[3] This section has been checked by Professor J. A. McGeoch, Editor, *Psychological Bulletin of the American Psychological Association.*

of the learning boundary conditions of which account will be taken here is *forgetting*. Enough good experimentation has been done in this field to make possible the statement of the following conclusions with some certainty. In an unoccupied mind (undisturbed sleep) practically all of the forgetting occurs during the first two hours. Interpolated mental activity that is not associated with the event or thing to be remembered accelerates forgetting. Interpolated mental activity dealing with things or events of a type remote from the event or thing to be remembered interferes least, while the higher the degree of similarity of the interpolated mental activity to the thing or event to be remembered, the greater the interference. A class in calculus or theoretical chemistry after a physics lesson will cause more forgetting than a class in English or history.

Forgetting is sometimes caused by a stimulation pattern that is altered from the one in the learning situation. An old friend in an unfamiliar suit of clothes or with a new mustache may be difficult to recognize. The *watt*, although a familiar term when found on the printed page with *foot pounds* and *horsepower*, might not be recognized when it appears with *volts* and *amperes*. *Young's modulus of elasticity* in the elementary physics laboratory is usually associated with a wire. It is not particularly surprising that this concept may not be recalled when it appears in connection with a beam in the mechanical engineering laboratory; the stimulation pattern connected with the concept has been extensively altered.

This brings up a third point to be suggested in connection with forgetting. Forgetting occurs when the individual acquires a changed point of view, a new set of values, a changed attitude or goal or a different frame of reference. This is a second reason for the student's not remembering about Young's modulus of elasticity. He learned it as a physics student and is asked to recall it as an engineer. The more meaningful the material and the better it is learned, the less rapidly one forgets. To be useful, material must be overlearned. Professors of engineering are continually saying that they cannot find a trace of evidence that junior engineers have ever had physics. For this situation, both the teachers of engineering and of physics are to blame. As has been pointed out, learning may be thought of as the formation of chains having many links. These learning chains, like actual metal chains, are as strong as their weakest link. Many times single links in long and important learning chains are broken and the whole chain becomes unusable. The teacher of engineering who says he finds "no evidence of physics" means that he has found no complete chains. However, if he looks for them, he will find many broken chains which might be easily and effectively repaired. Again, the "lack of evidence" may be due in part to the fact that the physics professor had not taught the student to identify these concepts in engineering situations. Also, all of the three factors that induce human forgetting have operated, particularly the last two—a changed stimulus-environment and a shifted point of view.

These things are mentioned because it is quite certain that the foregoing analysis of this deplorable situation is correct. When the causes are known and the mystery is taken out of a situation, it is usually much easier to do something intelligent about it.

## Motivation

The application of certain general principles of motivation can be depended on to produce results in engineering physics. One of the most powerful motives is *a feeling of achievement and of mastery*. The old saw, "nothing succeeds like success and nothing fails like failure," is literally true. The joy of achievement comes when one has done a job that is somewhere near to being commensurate with one's abilities. Applied to the classroom, this means keeping the student operating at the level of his ability, which in turn means plenty of labor for the teacher in outlining work that will enable the student to get going and keep going until the job at hand is done. It also means having small student sections of rather uniform ability. When new areas are approached, the work must be so graded in difficulty that the student will be able to get the first thing assigned. Success in the second may hinge on success in the first, and so on. If a student's mind is actually used in the way that he can use it, one has little to fear about his

interest, because the individual finds basic enjoyment in the use of his mind.

That this feeling of achievement may be based on a solid foundation, the student must be continually appraised of the state of his success. Students who are doing reasonably well are often discouraged because of long delay in returning written work. The student has a right to know at all times where he stands, and a knowledge of achievement produces more achievement.

A second powerful motive will operate when the student feels that he is getting what he needs and what will be valuable to him in his profession. Some salesmanship may have to be used to show him that his actual needs often are different from what he thinks they are; but, with the industrial and cultural applications of physics everywhere around us and with an ever increasing literature on the subject, that should not be a difficult task.

Another motivator is *the dramatization of physics*. The history of science is full of dramatic stories of scientific battles fought and won. Through them can be traced the growth of ideas. If this is carefully presented, a student will see that ideas evolve logically and seldom haphazardly, that ideas even now are growing and that he may well have a part in the development of some of them.

A powerful motive that can be used legitimately with some of the best students is that, other things being equal, there is *a large positive relationship between academic achievement and a good job at graduation*.

## Details of Objectives

*Objective 1.*—Four objectives have been listed. The first of these is a thorough mastery of the bases of physics. The student of liberal arts may get along very well with a passive knowledge of physics. Not so with the engineer; his knowledge of physics must be a genuine working knowledge. It is agreed that knowledge, to be genuinely useful, that is, to be working knowledge, must be overlearned. There is a great deal of difference between a conversational knowledge of a subject and ability to use that knowledge logically and effectively.

The first task for the physics teacher is to map out and set down the fundamentals in physics about which the engineer must have a working knowledge. These must form the central core of the course. The teacher must know something about the inherent difficulties of each of these fundamentals and consequently apportion the time of the course in accordance with the difficulties to be encountered. The textbook, lecturer, laboratory, recitation section and perhaps also a problem section are factors operating together and must share the responsibility of *getting the abstract principles out in the open, unattached to things*.

Experiments have shown that, for the student of average ability, this can usually be done by presenting illustrations of the principle, chosen from different fields and approaching the principle from as widely divergent angles as possible. For example, centrifugal force can be illustrated by a ball whirling on the end of a string, an automobile rounding a curve and a moving belt passing over a pulley. After these illustrations have been demonstrated experimentally and discussed with the aid of diagrams and equations, the average student can then be expected to recognize centrifugal force operating in an airplane looping the loop, a basket of eggs whirled over a child's head or the moon moving in its orbit. Measurements have shown that three illustrations as just indicated are adequate in the majority of cases.

After the principle has been abstracted, a demonstration should be given showing how it is identified and used in a problem situation. The student is then asked to use the principle in simple problems which he can easily solve, then in problems of increasing difficulty until he achieves mastery in most difficult situations. Finally, it is necessary to provide a maintenance program that will insure periodical reviews of this principle. At first, the review periods must not be too far separated in time. The more firmly the principle is established, the longer the time may be between review periods. If such a review maintenance program is systematically followed, the student will come to the end of the semester in full command of the things he has mastered.

That the laboratory, lectures, recitations and problem work should be carefully correlated is evidenced by results of many questionaries and observations carried out at various institutions, and is in accord with the best principles of learning. From the principles of learning and forgetting indicated in earlier sections it is quite clear that as short a period as possible should elapse between the time a principle is

first contacted in the textbook or the lecture and the time of its validation in the laboratory and use in problem-solving. This is one of the points at which many college courses in physics could be easily and effectively improved.

If a student is to be able to use physical concepts in actual situations, he must be taught to recognize these concepts when he encounters them in a new framework. To do this, the concept should be thoughtfully analyzed to bring its features into bold relief, and then practice should be afforded in identifying the concept in many and varied problems built around actual applications in actual situations.

Educational psychologists have given us very discouraging accounts of experiments in the transfer of learning, and experienced teachers know how woefully small is the transfer obtained when there is high expectation for more. A chemistry teacher once complained that after he teaches his students to do proportion with apples they cannot immediately do it with potatoes. Of course, most of them cannot, for the identification is tied up with apples. But if he taught them proportion in connection with apples, fence posts and atoms, quite likely they would be able to apply it to books, chocolate sodas and force components. This element of transfer is so vital that, as teachers, we can afford to pay a high price in class time for it. The approach pointed out is in harmony with the best modern educational psychology.

*Objective 2.*—The student should learn a great deal about the measuring instruments that physics has at its command and that are useful to the engineer, and should develop some skill in handling them. The whole laboratory situation needs so much careful consideration that no attempt will be made here to discuss it fully. Only a few remarks will be made.

About 50 years ago there came out of an eastern university a list of 40 physics laboratory experiments that are still probably the single most potent power operating in shaping our laboratory practice. If only we could by some magic shake ourselves loose from those 40 experiments and start over again! But much money has been invested in laboratory equipment which must be used, and the laboratory manuals have been written more or less around these 40 experiments which were designed before there were automobiles, radios, airplanes or x-ray machines. Can good reason be given as to why, in the laboratory part of engineering physics, the student should not measure the thermal conductivity of some of the modern insulating materials, photograph the spark spectrum of brass and compare it with similar spectra of copper and zinc, obtain data on the drag-lift ratio of an airplane wing section, make x-ray photographs of a steel weld or run the temperature viscosity curves for lubricating oils? Can there be doubt that these would stimulate interest?

*Objective 3.*—The student should develop a scientific attitude. Here I am reminded of a story. An artist, whose name I have forgotten, was exhibiting some of his paintings. One subject was a sunset over a Scottish heath. A lady remarked to the artist, "I have never seen a sunset like that," to which came the retort, courteously, "Don't you wish you could?"

The case for developing scientific attitude is not hopeless, however. Some things of interest can be said. Since 1933 a group of 30 high schools in various parts of the country have been carrying on a study of their curriculums with the assistance of a staff financed by the General Education Board. One aspect of the work has been the problem of the development of scientific attitudes. The best and most advanced technics of test-building have been used to devise methods of determining whether students could be found who were superior in the ability to interpret data, knew the necessary and sufficient conditions for proof, had an inner urge to seek new data about old unsolved problems, recognized scientific principles operating in everyday life, and evidenced interest in the "go of things." Such students *were* located, and it is not surprising that a higher percentage was found in some schools than in others. On examining the case histories of the students with best scientific attitudes, some definite things have been found. These studies are far from complete, and as yet are unpublished, but the findings so far appear to be about as follows:[4]

[4] Stated by permission of the director of the study, Professor Ralph Tyler, Department of Education, University of Chicago.

You are now becoming what you are going to be. A student who now knows how to interpret data is one who has been given practice in interpreting data. The one who recognizes scientific principles in everyday life is one who has been taught to identify textbook principles operating in everyday life, and rewarded for doing it. It appears to be as simple—or as difficult—as that.

*Objective 4.*—A course in engineering physics should kindle the imagination and fire the creative abilities of the student. Concerning this objective it may be remarked again that "one is now becoming what he is going to be"; and if the imagination is to be kindled and the creative abilities developed, then these things must be practiced and a student must not be punished every time his creative instincts lead him away from the beaten path. He must be given opportunity and encouragement to follow leads and to interpret the results obtained from them. May it also be added that contact in the classroom and laboratory with a *creative* teacher will probably not hinder the development of these desirable traits.

To teach in the way suggested would be expensive in money and in teaching time. If the reader thinks that the administrative officers of his institution would be interested in having him spend time and money that way, the project is worth a trial, and I will predict decisive results.

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# The Collision of Two Particles: Relativistic Collisions; Collisions Between Quanta and Material Particles

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## II. Relativistic Collisions

IN Part I,[10] there was outlined a brief summary of the principal facts concerning collisions between two particles whose velocities are small enough to use ordinary classical (Newtonian) mechanics. In this section, we shall outline the corresponding laws governing high velocity collisions, using the principles of restricted relativity. Practical interest in such collisions is aroused by the work of Champion and his collaborators[11] who have observed such encounters in the cloud chamber.

The following outline of the fundamentals of restricted relativity[12] is given without proof; it is intended to refresh the reader's memory and also to introduce the notation we shall use in discussing collisions.

These symbols will have the same meaning as in Part I:

$m$ = rest mass of first particle,
$\mathbf{a}$ = velocity of this particle before impact,
$\mathbf{u}$ = velocity of this particle after impact,
$n$ = rest mass of second particle,
$\mathbf{b}$ = velocity of this particle before impact,
$\mathbf{v}$ = velocity of this particle after impact.

Consider two observers, the second moving with velocity $\mathbf{V}$ with respect to the first. We shall denote quantities as observed by the first observer by the subscript $_s$; those observed by the second by the subscript $_g$. For convenience in printing, we shall write the Lorentz factor as $c/(c^2-V^2)^{\frac{1}{2}}$. Then, if $L$ be the length of a rod when measured by an observer at rest with it, its apparent length to another observer, if it move with velocity $u$ parallel to its own length, is

$$L_s = L(c^2-u_s^2)^{\frac{1}{2}}/c, \quad L_g = L(c^2-u_g^2)^{\frac{1}{2}}/c. \tag{16}$$

Similarly, if its mass as it appears to an observer at rest be $m$, and its total energy be $W$, other observers will find

$$\begin{aligned} m_s &= mc/(c^2-u_s^2)^{\frac{1}{2}}, \quad & m_g &= mc/(c^2-u_g^2)^{\frac{1}{2}}, \\ W_s &= Wc/(c^2-u_s^2)^{\frac{1}{2}}, \quad & W_g &= Wc/(c^2-u_g^2)^{\frac{1}{2}}, \end{aligned} \tag{17}$$

where $W=mc^2$, $W_s=m_sc^2$ and $W_g=m_gc^2$. Its

[10] R. H. Bacon, Am. J. Phys. **8**, 154 (1940). Numbering of references, figures and equations continued from Part I.
[11] F. C. Champion, Proc. Roy. Soc. **A136**, 630 (1932); **A137**, 688 (1932).
[12] For a rigorous derivation and complete interpretation of Eqs. (16) to (26), see, for example, Tolman, *Relativity, thermodynamics and cosmology* (Oxford, 1934), pp. 1–58.

kinetic energy will be

$$E_s = W_s - W = mc^2\left[\frac{c}{(c^2-u_s^2)^{\frac{1}{2}}}-1\right],$$
$$E_o = W_o - W = mc^2\left[\frac{c}{(c^2-u_o^2)^{\frac{1}{2}}}-1\right]. \tag{18}$$

In the limiting case of small velocities, these expressions reduce to the familiar classical ones.

The momentum of a moving object is defined as follows:

$$\mathbf{M}_s = m_s\mathbf{u}_s = mc\mathbf{u}_s/(c^2-u_s^2)^{\frac{1}{2}} = mc\mathbf{n}_s, \tag{19}$$
$$\mathbf{M}_o = m_o\mathbf{u}_o = mc\mathbf{u}_o/(c^2-u_o^2)^{\frac{1}{2}} = mc\mathbf{n}_o,$$

where the quasi velocities $\boldsymbol{\alpha}_s$, $\boldsymbol{\beta}_s$, $\mathbf{n}_s$, $\mathbf{v}_s$, $\boldsymbol{\alpha}_o$, $\boldsymbol{\beta}_o$, $\mathbf{n}_o$ and $\mathbf{v}_o$ are defined as follows:

$$\begin{aligned}
\boldsymbol{\alpha}_s &= \mathbf{a}_s/(c^2-a_s^2)^{\frac{1}{2}}, & \boldsymbol{\alpha}_o &= \mathbf{a}_o/(c^2-a_o^2)^{\frac{1}{2}},\\
\boldsymbol{\beta}_s &= \mathbf{b}_s/(c^2-b_s^2)^{\frac{1}{2}}, & \boldsymbol{\beta}_o &= \mathbf{b}_o/(c^2-b_o^2)^{\frac{1}{2}},\\
\mathbf{n}_s &= \mathbf{u}_s/(c^2-u_s^2)^{\frac{1}{2}}, & \mathbf{n}_o &= \mathbf{u}_o/(c^2-u_o^2)^{\frac{1}{2}},\\
\mathbf{v}_s &= \mathbf{v}_s/(c^2-v_s^2)^{\frac{1}{2}}, & \mathbf{v}_o &= \mathbf{v}_o/(c^2-v_o^2)^{\frac{1}{2}}.
\end{aligned} \tag{20}$$

These quantities are more convenient to use, in many cases, than the true velocities.

Having defined the energy and momentum of a particle, we can write the fundamental relativistic energy-momentum relation,

$$W_s^2 = W^2 + M_s^2c^2, \quad W_o^2 = W^2 + M_o^2c^2. \tag{21}$$

If $T$ be an interval measured by a clock at rest with respect to some observer, the same interval will appear to our observers as

$$T_s = cT/(c^2-u_s^2)^{\frac{1}{2}}, \quad T_o = cT/(c^2-u_o^2)^{\frac{1}{2}}. \tag{22}$$

Now, let the coordinates used by the first observer be $x_s$, $y_s$ and $z_s$; those used by the second observer, $x_o$, $y_o$ and $z_o$; let the two frames of reference coincide at the time $t_s = t_o = 0$; and let $\mathbf{V}$ be parallel to the $x$ direction. Then, the coordinates of a point, as seen by one observer, will be related to those as seen by the other as follows:

$$\begin{aligned}
x_s &= c(x_o + Vt_o)/(c^2-V^2)^{\frac{1}{2}},\\
x_o &= c(x_s - Vt_s)/(c^2-V^2)^{\frac{1}{2}},\\
y_s &= y_o, \quad z_s = z_o,\\
t_s &= (c^2t_o + x_oV)/[c(c^2-V^2)^{\frac{1}{2}}],\\
t_o &= (c^2t_s - x_sV)/[c(c^2-V^2)^{\frac{1}{2}}].
\end{aligned} \tag{23}$$

Since each observer measures length and time differently, their measures of velocity will be different also. Differentiation of Eqs. (23) gives us

$$\begin{aligned}
u_{sx} &= c^2(u_{ox}+V)/(c^2+\mathbf{u}_o\cdot\mathbf{V}),\\
u_{sy} &= u_{oy}c(c^2-V^2)^{\frac{1}{2}}/(c^2+\mathbf{u}_o\cdot\mathbf{V}),\\
u_{sz} &= u_{oz}c(c^2-V^2)^{\frac{1}{2}}/(c^2+\mathbf{u}_o\cdot\mathbf{V}),\\
u_{ox} &= c^2(u_{sx}-V)/(c^2-\mathbf{u}_s\cdot\mathbf{V}),\\
u_{oy} &= u_{sy}c(c^2-V^2)^{\frac{1}{2}}/(c^2-\mathbf{u}_s\cdot\mathbf{V}),\\
u_{oz} &= u_{sz}c(c^2-V^2)^{\frac{1}{2}}/(c^2-\mathbf{u}_s\cdot\mathbf{V}),
\end{aligned} \tag{24}$$

where $u_{sx} = dx_s/dt_s$, $u_{ox} = dx_o/dt_o$,
$u_{sy} = dy_s/dt_s$, $u_{oy} = dy_o/dt_o$,
$u_{sz} = dz_s/dt_s$, $u_{oz} = dz_o/dt_o$.

Knowing the velocities as measured by one observer in terms of those as seen by the other, we can now derive some auxiliary relations:

$$\begin{aligned}
c^2-u_s^2 &= c^2(c^2-u_o^2)(c^2-V^2)/(c^2+\mathbf{u}_o\cdot\mathbf{V})^2,\\
c^2-u_o^2 &= c^2(c^2-u_s^2)(c^2-V^2)/(c^2-\mathbf{u}_s\cdot\mathbf{V})^2;
\end{aligned} \tag{25}$$

$$m_s = m_o\frac{c^2+\mathbf{u}_o\cdot\mathbf{V}}{c(c^2-V^2)^{\frac{1}{2}}}, \quad m_o = m_s\frac{c^2-\mathbf{u}_s\cdot\mathbf{V}}{c(c^2-V^2)^{\frac{1}{2}}}; \tag{26}$$

$$\begin{aligned}
M_{sx} &= [c/(c^2-V^2)^{\frac{1}{2}}](M_{ox}+m_oV),\\
M_{sy} &= M_{oy}, \quad M_{sz} = M_{oz},\\
M_{ox} &= [c/(c^2-V^2)^{\frac{1}{2}}](M_{sx}-m_sV);
\end{aligned} \tag{27}$$

$$\begin{aligned}
\eta_{sx} = u_{sx}/(c^2-u_s^2)^{\frac{1}{2}} &= [c/(c^2-V^2)^{\frac{1}{2}}]\eta_{ox}\\
&\quad + [V/(c^2-V^2)^{\frac{1}{2}}](1+\eta_o^2)^{\frac{1}{2}},\\
\eta_{sy} = \eta_{oy}, \quad \eta_{sz} &= \eta_{oz},\\
\eta_{ox} = u_{ox}/(c^2-u_o^2)^{\frac{1}{2}} &= [c/(c^2-V^2)^{\frac{1}{2}}]\eta_{sx}\\
&\quad - [V/(c^2-V^2)^{\frac{1}{2}}](1+\eta_s^2)^{\frac{1}{2}}.
\end{aligned} \tag{28}$$

So much for the description of the dynamics of a single particle. In what follows, we shall use $\mathbf{M}_s$ and $\mathbf{M}_o$ to denote the sum of the momentums of the colliding particles, and $W_s$ and $W_o$ to denote the sum of their energies, so that our equations of motion, corresponding to Eqs. (1) and (2) in Part I, become

$$\begin{aligned}
\mathbf{M}_s &= c[m\mathbf{a}_s/(c^2-a_s^2)^{\frac{1}{2}} + n\mathbf{b}_s/(c^2-b_s^2)^{\frac{1}{2}}]\\
&= c[m\mathbf{u}_s/(c^2-u_s^2)^{\frac{1}{2}} + n\mathbf{v}_s/(c^2-v_s^2)^{\frac{1}{2}}],
\end{aligned} \tag{1r}$$

$$\begin{aligned}
W_s &= c^3[m/(c^2-a_s^2)^{\frac{1}{2}} + n/(c^2-b_s^2)^{\frac{1}{2}}]\\
&= c^3[m/(c^2-u_s^2)^{\frac{1}{2}} + n/(c^2-v_s^2)^{\frac{1}{2}}].
\end{aligned} \tag{2r}$$

In terms of the quasi velocities, these are

$$\mathbf{M}_s = c(m\boldsymbol{\alpha}_s + n\boldsymbol{\beta}_s) = c(m\mathbf{n}_s + n\mathbf{v}_s), \tag{1r'}$$

$$\begin{aligned}
W_s &= c^2[m(1+\alpha_s^2)^{\frac{1}{2}} + n(1+\beta_s^2)^{\frac{1}{2}}]\\
&= c^2[m(1+\eta_s^2)^{\frac{1}{2}} + n(1+v_s^2)^{\frac{1}{2}}].
\end{aligned} \tag{2r'}$$

We cannot treat inelastic relativistic collisions in the same fashion as the classical ones; we

must know the origin or the fate of the energy difference, for the energy difference will possess mass and momentum. If, for example, this energy difference is a mere change in the heat of one or the other of the colliding bodies (or of both), then the rest mass of the particle concerned is changed, the total energy $W$ of the system is unchanged and the interaction remains a two-body collision.

The velocity of the center of mass is

$$\mathbf{V}_s=\mathbf{M}_s/(m_s+n_s)=\mathbf{M}_sc^2/W_s. \qquad (3r)$$

Now, since the two observers measure mass, position and velocity differently, it is not immediately obvious that an observer traveling with velocity $\mathbf{V}_s$ will agree with the first observer as to the location and motion of the center of mass as seen by him. Therefore, let us choose an observer who does find the center of mass of the system to be stationary with respect to himself, and let his velocity with respect to the first observer be $\mathbf{V}_g$. Then, from Eqs. (29),

$$M_{gx}=0=[c/(c^2-V_g^2)^{\frac{1}{2}}][M_{sx}-(m_s+n_s)V_g],$$
$$M_{gy}=M_{sy}=M_{gz}=M_{sz}=0,$$

giving

$$\mathbf{M}_g=[c/(c^2-V_g^2)^{\frac{1}{2}}][\mathbf{M}_s-(m_s+n_s)\mathbf{V}_g]=0,$$

and hence

$$\mathbf{V}_g=\mathbf{M}_s/(m_s+n_s)=\mathbf{V}_s=\mathbf{V}.$$

Therefore, while the two observers may not agree as to the exact location of the center of mass at any instant, they do agree as to its motion with respect to the first observer, and so we can drop the subscript on $\mathbf{V}$ and use the equations so far developed.

Combining Eqs. (25), (26) and (27), we easily obtain

$$W_s=W_gc/(c^2-V^2)^{\frac{1}{2}},$$
$$M_sc=W_gV/(c^2-V^2)^{\frac{1}{2}}, \qquad (4r)$$
$$W_s^2-M_s^2c^2=W_g^2.$$

Eqs. (4r) are the relativistic analogs of Eq. (4) of Part I, which states the relation between the kinetic energy in the center-of-mass system and the kinetic energy and momentum observed in any other system. In addition, Eqs. (4r) hold for any number of particles, and the third equation of this set is merely the fundamental energy-momentum relation, Eq. (21), restated so as to apply to a system of particles.

We shall use the symbols $J$, $K$ and $L$ as abbreviations for the following quantities which occur frequently:

$$J=W_s^2-M_s^2c^2+(m^2-n^2)c^4,$$
$$K=W_s^2-M_s^2c^2+(n^2-m^2)c^4,$$
$$J^2-4W_g^2m^2c^4=K^2-4W_g^2n^2c^4=L^2.$$

No subscript is needed on these symbols, as the quantities they represent have the same values for all observers.

We shall number our equations to agree with the corresponding classical ones in Part I.

### (G) Collinear Collisions

Here we can replace the vectors by their scalar magnitudes, so that

$$ma_s/(c^2-a_s^2)^{\frac{1}{2}}+nb_s/(c^2-b_s)^{\frac{1}{2}}=mu_s/(c^2-u_s^2)^{\frac{1}{2}}+nv_s/(c^2-v_s^2)^{\frac{1}{2}}, \qquad (1ra)$$

$$m/(c^2-a_s^2)^{\frac{1}{2}}+n/(c^2-b_s^2)^{\frac{1}{2}}=m/(c^2-u_s^2)^{\frac{1}{2}}+n/(c^2-v_s^2)^{\frac{1}{2}}. \qquad (2ra)$$

To solve these for $u$ and $v$ in terms of $m$, $n$, $a$ and $b$, while easy enough, is not very useful, as the resulting expressions are long and unwieldy. A much simpler procedure is the following:

$$u_s=c^2(u_g+V)/(c^2+u_gV), \quad u_g=-a_g,$$
$$a_g=c^2(a_s-V)/(c^2-a_sV),$$

therefore

$$u_s=[c^2(2V-a_s)-a_sV^2]/(c^2+V^2-2a_sV). \qquad (6ra)$$

In the limiting case of small velocities, this reduces to $u_s=2V-a_s$, a relation obtained by applying the foregoing reasoning to the classical case; it may also be obtained from Eq. (6a). Substituting Eq. (3r) into (6ra), we have

$$u_s=\frac{M_sc^2(2W_s-M_sa_s)-a_sW_s^2}{W_s^2+M_s^2c^2-2M_sa_sW_s}. \qquad (6ra')$$

Since $M_s$, $W_s$ and $a_s$ are known to begin with, we can now find $u_s$. This is probably the simplest expression obtainable for $u_s$. The corresponding equation for $v_s$ is obvious.

Solving the equations analogous to Eqs. (1c) and (2c) in Part I,

$$M_s=c[mu_s/(c^2-u_s^2)^{\frac{1}{2}}+nv_s/(c^2-v_s^2)^{\frac{1}{2}}], \qquad (1rc)$$

$$W_s=c^2[m/(c^2-u_s^2)^{\frac{1}{2}}+n/(c^2-v_s^2)^{\frac{1}{2}}], \qquad (2rc)$$

we obtain

$$u_s = c(4W_s M_s c m^2 c^4 \pm JL)/(J^2 + 4m^2c^4 M_s^2 c^2), \quad (6rc)$$

$$v_s = c(4W_s M_s c n^2 c^4 \mp KL)/(K^2 + 4n^2c^4 M_s^2 c^2), \quad (7rc)$$

where the two upper signs go together, and where the meaning is the same as in Eqs. (6c) and (7c): if one pair of signs tells what happened after the collision, the other tells what happened before.

It is somewhat easier to use the quasi velocities. Then our equations corresponding to Eqs. (1a) and (2a) of Part I become

$$m\alpha_s + n\beta_s = m\eta_s + n\upsilon_s, \quad (1r'a)$$

$$m(1+\alpha_s^2)^{\frac{1}{2}} + n(1+\beta_s^2)^{\frac{1}{2}} = m(1+\eta_s^2)^{\frac{1}{2}} + n(1+\upsilon_s^2)^{\frac{1}{2}}. \quad (2r'a)$$

The solutions are

$$\eta_s = \frac{2mn\beta_s + (m^2-n^2)\alpha_s + 2n^2\beta_s\{[(1+\alpha_s^2)(1+\beta_s^2)]^{\frac{1}{2}} - \alpha_s\beta_s\}}{m^2+n^2+2mn\{[(1+\alpha_s^2)(1+\beta_s^2)]^{\frac{1}{2}} - \alpha_s\beta_s\}} \quad (6r'a)$$

and a similar expression for $\upsilon_s$. Then, since

$$u_s = \eta_s c/(1+\eta_s^2)^{\frac{1}{2}},$$

we can consider the equations solved for $u_s$ and $v_s$ also. These equations have the advantage that they reduce to the form of Eqs. (6a) and (7a) in the limit of very small velocities. However, $\eta_s$ is a pure number, not a velocity, and such similarities should be used with caution.

It is easily shown that the quantity

$$[(1+\alpha_s^2)(1+\beta_s^2)]^{\frac{1}{2}} - \boldsymbol{\alpha}_s \cdot \boldsymbol{\beta}_s = (c^2 - \mathbf{a}_s \cdot \mathbf{b}_s)/[(c^2-a_s^2)(c^2-b_s^2)]^{\frac{1}{2}}$$

is the same for all observers; that is,

$$[(1+\alpha_\sigma^2)(1+\beta_\sigma^2)]^{\frac{1}{2}} - \boldsymbol{\alpha}_\sigma \cdot \boldsymbol{\beta}_\sigma = [(1+\alpha_s^2)(1+\beta_s^2)]^{\frac{1}{2}} - \boldsymbol{\alpha}_s \cdot \boldsymbol{\beta}_s,$$

or

$$(c^2 - \mathbf{a}_\sigma \cdot \mathbf{b}_\sigma)/[(c^2-a_\sigma^2)(c^2-b_\sigma^2)]^{\frac{1}{2}} = (c^2 - \mathbf{a}_s \cdot \mathbf{b}_s)/[(c^2-a_s^2)(c^2-b_s^2)]^{\frac{1}{2}}.$$

Since this is the same for all observers, it is likewise the same for the observer moving with one of the particles, so that it enables one to find the relative speed of one of the particles with respect to the other. Further, the quantity is the same before and after the collision if the total energy $W_s$ is unchanged, so that we may write

$$[(1+\alpha_s^2)(1+\beta_s^2)]^{\frac{1}{2}} - \boldsymbol{\alpha}_s \cdot \boldsymbol{\beta}_s = [(1+\eta_s^2)(1+\upsilon_s^2)]^{\frac{1}{2}} - \mathbf{n}_s \cdot \boldsymbol{\upsilon}_s,$$
$$(c^2 - \mathbf{a}_s \cdot \mathbf{b}_s)/[(c^2-a_s^2)(c^2-b_s^2)]^{\frac{1}{2}} = (c^2 - \mathbf{u}_s \cdot \mathbf{v}_s)/[(c^2-u_s^2)(c^2-v_s^2)]^{\frac{1}{2}}$$

as a relativistic analog of Eq. (5a) in Part I.

Given the equations

$$M_s = c(m\eta_s + n\upsilon_s), \quad (1r'c)$$

$$W_s = c^2[m(1+\eta_s^2)^{\frac{1}{2}} + n(1+\upsilon_s^2)^{\frac{1}{2}}], \quad (2r'c)$$

the solutions are

$$\eta_s = (JM_s c \pm W_s L)/2mc^2(W_s^2 - M_s^2c^2), \quad (6r'c)$$

$$\upsilon_s = (KM_s c \mp W_s L)/2nc^2(W_s^2 - M_s^2c^2), \quad (7r'c)$$

where again the two upper signs go together, and the meaning is the same as in Eqs. (6c) and (7c). It is obvious that the momentums and the energies, and also the observed masses, are more easily obtained from Eqs. (6r'c) and (7r'c) than from (6rc) and (7rc).

If $m=n$ (if the *rest* masses are alike), the particles merely exchange velocities in a collinear elastic collision, as in the classical case.

### (H) Oblique Collisions

This section corresponds to Section (E) in Part I. The equations for the most general type of

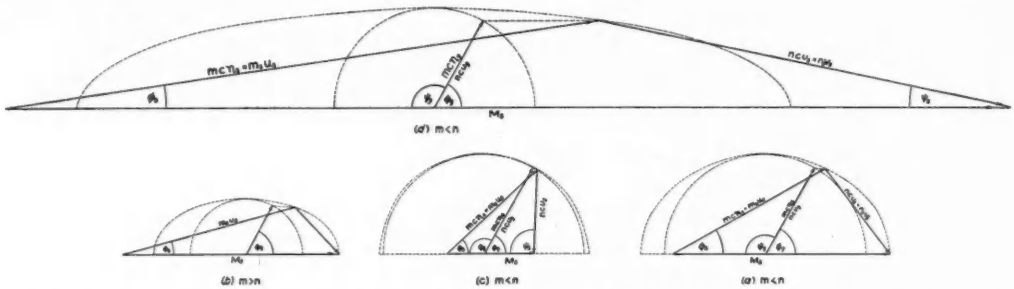

FIG. 6. Momentum relations in elastic collisions between particles of very high velocity, obeying relativistic mechanics: (*a*) relativistic analog of Fig. 1(*a*), Part I; the more massive particle is at rest in the observer's frame before the collision; (*b*) relativistic analog of Fig. 1(*c*); the less massive particle is initially at rest; (*c*) and (*d*), same collision as that of Fig. 6(*a*), but as seen by two other observers; these are analogs of Figs. 3(*a*) and 3(*c*), but with the ratio of the masses reversed. $\phi_\theta$ and $\psi_\theta$ are the angles between the final directions of motion, as observed in the center-of-mass system, and the direction of motion of the center of mass with respect to the other observers; $\phi_s$ and $\psi_s$ are the angles between the final motions (with respect to the observer's frame) and $\mathbf{M}_s$. The radii of the circles are the momentums of the individual particles as observed in the center-of-mass system. The triangles represent the relation $\mathbf{M}_s=m_s\mathbf{u}_s+n_s\mathbf{v}_s=$const. The centers of the circles (and therefore, of the ellipses) are at the points dividing $\mathbf{M}_s$ in the ratio $m/n$. The relation between $\phi_\theta$ and $\phi_s$ (or between $\psi_\theta$ and $\psi_s$) is easily determined graphically from these diagrams by using Eqs. (28) or (27) for the $y$-components, as shown by the horizontal dotted lines.

collision involving only two masses are:

$$\mathbf{M}_s=c\left[\frac{m\mathbf{u}_s}{(c^2-u_s^2)^{\frac{1}{2}}}+\frac{n\mathbf{v}_s}{(c^2-v_s^2)^{\frac{1}{2}}}\right], \tag{1re}$$

$$W_s=c^3\left[\frac{m}{(c^2-u_s^2)^{\frac{1}{2}}}+\frac{n}{(c^2-v_s^2)^{\frac{1}{2}}}\right]. \tag{2re}$$

Now, resolve $\mathbf{u}_s$ into two components parallel and perpendicular to $\mathbf{M}_s$, and $\mathbf{v}_s$ into three rectangular components, as we have done in the classical cases before. Then our equations become

$$\begin{aligned}
M_s&=c[mp/(c^2-p^2-q^2)^{\frac{1}{2}}+nr/(c^2-r^2-s^2-t^2)^{\frac{1}{2}}],\\
0&=c[mq/(c^2-p^2-q^2)^{\frac{1}{2}}-ns/(c^2-r^2-s^2-t^2)^{\frac{1}{2}}],\\
0&=c[0+nt/(c^2-r^2-s^2-t^2)^{\frac{1}{2}}],\\
W_s&=c^3[m/(c^2-p^2-q^2)^{\frac{1}{2}}+n/(c^2-r^2-s^2-t^2)^{\frac{1}{2}}],
\end{aligned} \tag{7re}$$

from which we see that $\mathbf{M}_s$, $\mathbf{u}_s$ and $\mathbf{v}_s$ must be coplanar. Dropping $t$ and eliminating $r$ and $s$, we obtain $J(c^2-p^2-q^2)^{\frac{1}{2}}=2mc^3(W_s-M_sp)$. But since $p=u_s\cos\phi_s$ and $p^2+q^2=u_s^2$, this becomes

$$J(c^2-u_s^2)^{\frac{1}{2}}=2mc^3(W_s-M_su_s\cos\phi_s). \tag{14re}$$

Solving for $u_s$, we have

$$u_s=\frac{c[4W_sM_scm^2c^4\cos\phi_s\pm J(L^2-4m^2c^4M_s^2c^2\sin^2\phi_s)^{\frac{1}{2}}]}{J^2+4m^2c^4M_s^2c^2\cos^2\phi_s}. \tag{6re}$$

Similarly, for $v_s$, we obtain

$$v_s=\frac{c[4W_sM_scn^2c^4\cos\psi_s\mp K(L^2-4n^2c^4M_s^2c^2\sin^2\psi_s)^{\frac{1}{2}}]}{K^2+4n^2c^4M_s^2c^2\cos^2\psi_s}. \tag{7re}$$

These are the polar equations of ellipses whose centers are at

$$\left(\frac{c4W_sM_scm^2c^4}{J^2+4m^2c^4M_s^2c^2},\ 0\right) \quad\text{and}\quad \left(\frac{c4W_sM_scn^2c^4}{K^2+4n^2c^4M_s^2c^2},\ 0\right)$$

and whose semiaxes are

$$cJL/(J^2+4m^2c^4M_s^2c^2) \quad \text{and} \quad cJL/[J(J^2+4m^2c^4M_s^2c^2)^{\frac{1}{2}}]$$

for one ellipse, and

$$cKL/(K^2+4n^2c^4M_s^2c^2) \quad \text{and} \quad cKL/[K(K^2+4n^2c^4M_s^2c^2)^{\frac{1}{2}}]$$

for the other. They are illustrated in Fig. 7.

The relation between $\phi_s$ and $\psi_s$ is rather long to write out. The method of obtaining it is described in Section (I).

In terms of the quasi velocities, the equations are

$$\mathbf{M}_s=c(m\mathbf{n}_s+n\mathbf{v}_s), \tag{1r'e}$$

$$W_s=c^2[m(1+\eta_s^2)^{\frac{1}{2}}+n(1+v_s^2)^{\frac{1}{2}}]. \tag{2r'e}$$

Now, using the same method as outlined in Section (E), Part I, let

$$\mathbf{n}_s=p\mathbf{M}_s+q\mathbf{j},$$
$$\mathbf{v}_s=r\mathbf{M}_s-s\mathbf{j}+t\mathbf{k},$$

giving $c(mp+nr)=1$, $mq-ns=0$, $t=0$. Proceeding as before, we get

$$W_s=c^2\{m[1+M_s^2(p^2+q^2)]^{\frac{1}{2}}+n[1+M_s^2(r^2+s^2)]^{\frac{1}{2}}\}.$$

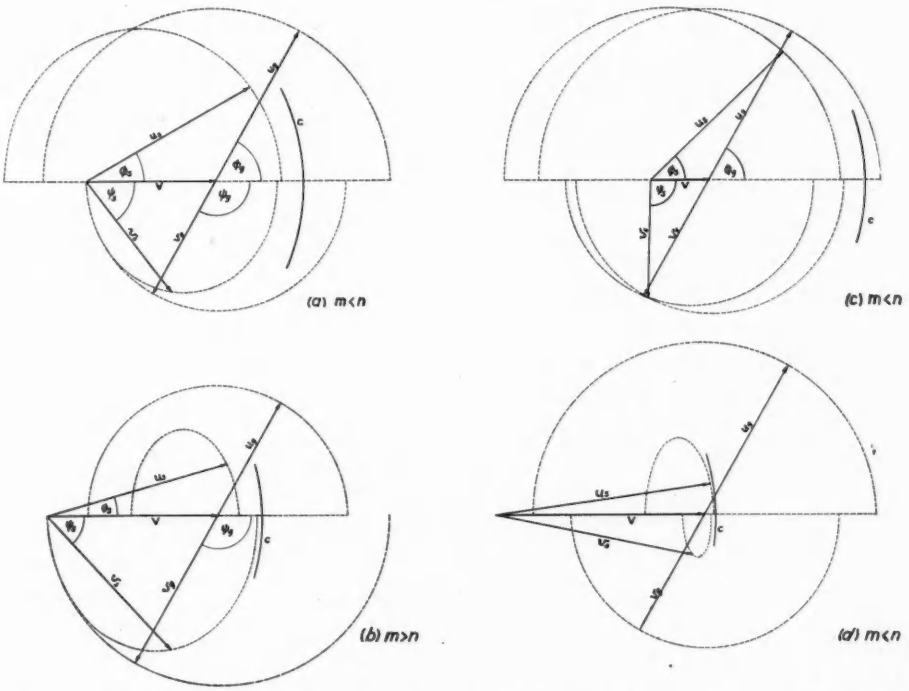


FIG. 7. Velocity relations in the same collisions as those depicted in Fig. 6. Here, the radii of the circles are the velocities as seen in the center-of-mass coordinates. The arc $c$ (solid line) represents the limiting velocity with respect to the observer (the velocity of light). $\mathbf{V}$ is the velocity of the center of mass with respect to the observer. Having obtained $\phi_s$ and $\psi_s$ graphically from Fig. 6, one can then obtain $\mathbf{u}_s$ and $\mathbf{v}_s$ graphically from Fig. 7.

Elimination of $r$ and $s$ results in

$$2W_s[m^2c^4+M_s^2m^2c^4(p^2+q^2)]^{\frac{1}{2}}=J+2mc^2M_s^2cp.$$

Remembering that $M_s^2(p^2+q^2)=\eta_s^2$, $pM_s=\eta_s\cos\phi_s$, we obtain

$$\eta_s=\frac{JM_sc\cos\phi_s\pm W_s(L^2-4m^2c^4M_s^2c^2\sin^2\phi_s)^{\frac{1}{2}}}{2mc^2(W_s^2-M_s^2c^2\cos^2\phi_s)}, \tag{6r'e}$$

$$v_s=\frac{KM_sc\cos\psi_s\mp W_s(L^2-4n^2c^4M_s^2c^2\sin^2\psi_s)^{\frac{1}{2}}}{2nc^2(W_s^2-M_s^2c^2\cos^2\psi_s)}. \tag{7r'e}$$

These are the equations of ellipses whose centers are at

$$(JM_sc/2mc^2(W_s^2-M_s^2c),\ 0) \quad \text{and} \quad (KM_sc/2nc^2[W_s^2-M_s^2c^2],\ 0),$$

and whose semiaxes are

$$W_sL/2mc^2(W_s^2-M_s^2c^2) \quad \text{and} \quad L/2mc^2(W_s^2-M_s^2c^2)^{\frac{1}{2}}$$

for one ellipse, and

$$W_sL/2nc^2(W_s^2-M_s^2c^2) \quad \text{and} \quad L/2nc^2(W_s^2-M_s^2c^2)^{\frac{1}{2}}$$

for the other. They are illustrated in Fig. 6.

### (I) The Center-of-Mass Coordinate System

This corresponds to Section (F) in Part **I**. In relativistic mechanics it is not as easy to transpose from the center-of-mass system to some other system as it is in Newtonian mechanics. Nevertheless, we shall outline the transposition for the sake of completeness. Remembering that $J$, $K$ and $L$ have the same values for all observers, we have, for an elastic impact,

$$u_g=cL/J=a_g, \tag{6rf}$$

$$v_g=cL/K=b_g, \tag{7rf}$$

$$\eta_g=L/2mc^2W_g=\alpha_g, \tag{6r'f}$$

$$v_g=L/2nc^2W_g=\beta_g. \tag{7r'f}$$

These are the equations of the circles shown in Figs. 6 and 7.

Referring back to Eqs. (24), we have

$$u_{gx}=c^2(u_{sx}-V)/(c^2-\mathbf{u}_s\cdot\mathbf{V}),$$
$$u_{gy}=cu_{sy}(c^2-V^2)^{\frac{1}{2}}/(c^2-\mathbf{u}_s\cdot\mathbf{V}).$$

Squaring and adding these, and solving the resulting equation for $u_s$, we get

$$u_s=\frac{c^2[(c^2-u_g^2)V\cos\phi_s\pm[(c^2-V^2)\{(c^2-u_g^2)V^2\cos^2\phi_s+c^2(u_g^2-V^2)\}]^{\frac{1}{2}}]}{c^2(c^2-V^2)+(c^2-u_g^2)V^2\cos^2\phi_s}. \tag{6rf'}$$

Combining this with Eqs. (3r), (4r) and (6rf), we can obtain Eq. (6re). In a similar manner, we can obtain Eq. (7re).

Equation (6rf') is the equation of the ellipse whose center is at

$$\left(\frac{Vc^2(c^2-u_g^2)}{c^4-u_g^2V^2},\ 0\right)$$

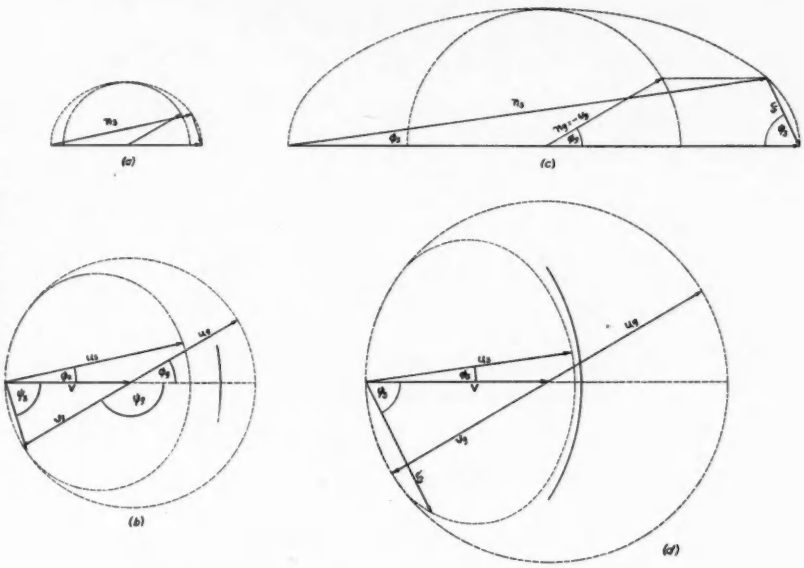

FIG. 8. (*a*), Momentum relations in the elastic collision between an electron initially at rest and one whose kinetic energy is 0.51 Mev (its observed mass is twice its rest mass); (*b*), velocity relations in the same collision; (*c*) and (*d*), the case in which the kinetic energy of the incident electron is 2.55 Mev (the observed mass is six times the rest mass). These are relativistic analogs of Figs. 1(*b*) and 2(*b*), Part I.

and whose semiaxes are $u_g c^2(c^2-V^2)/(c^4-u_g^2V^2)$ and $u_g c(c^2-V^2)^{\frac{1}{2}}/(c^4-u_g^2V^2)^{\frac{1}{2}}$. It is, of course, the same ellipse as that given by Eq. (6re).

Similarly, for the quasi velocities, we obtain from Eqs. (28) the relation

$$\eta_s=\frac{[(c^2-V^2)(1+\eta_g^2)]^{\frac{1}{2}}\cos\phi_s\pm[\eta_g^2(c^2-V^2)-V^2\sin^2\phi_s]^{\frac{1}{2}}}{c^2-V^2\cos^2\phi_s}. \tag{6r'f'}$$

Combination of this with Eqs. (3r), (4r) and (6r'f) will give Eq. (6r'e).

Equation (6r'f') is the equation of the ellipse with its center at

$$\left(\frac{V}{(c^2-V^2)^{\frac{1}{2}}}(1+\eta_g^2)^{\frac{1}{2}},\ 0\right)$$

and with semiaxes

$$\eta_g c/(c^2-V^2)^{\frac{1}{2}} \text{ and } \eta_g,$$

which is, of course, the same ellipse as that given by Eq. (6r'e). From this last result we see that this ellipse is similar to that given by Eq. (7r'e).

The process of obtaining the relation between $\phi_s$ and $\psi_s$ is exactly analogous to that of obtaining Eqs. (15d) and (15e) in the classical case, Part I, Section (F). Referring to Fig. 7, we see that

$$\cot\phi_g=\eta_{gx}/\eta_{gy}=-\cot\psi_g=-v_{gx}/v_{gy}.$$

From Eqs. (6r'f) and the definition of $L$ we have

$$\eta_g=L/2mc^2W_g \quad \text{and} \quad (1+\eta_g^2)^{\frac{1}{2}}=J/2mc^2W_g.$$

From Eq. (4r), we obtain

$$c/(c^2-V^2)^{\frac{1}{2}}=W_s/W_g \quad \text{and} \quad V/(c^2-V^2)^{\frac{1}{2}}=M_s c/W_g.$$

Combination of these with Eqs. (28) gives

$$\cot \phi_s = \eta_{sz}/\eta_{sy} = (W_s \cot \phi_g/W_g) + (M_s cJ \csc \phi_g/W_g L),$$
$$\cot \psi_s = v_{sz}/v_{sy} = (-W_s \cot \phi_g/W_g) + (M_s cK \csc \phi_g/W_g L).$$

Addition and subtraction of these yields

$$L(\cot \phi_s + \cot \psi_s) = \frac{M_s c}{W_g}(J+K) \csc \phi_g = 2W_g M_s c \csc \phi_g,$$
$$\frac{M_s c}{W_s}(K \cot \phi_s - J \cot \psi_s) = \frac{M_s c}{W_g}(J+K) \cot \phi_g = 2W_g M_s c \cot \phi_g.$$

Squaring each of these, subtracting and reducing, we have

$$(W_s^2 L^2 - M_s^2 c^2 K^2) \cot^2 \phi_s + (W_s^2 L^2 - M_s^2 c^2 J^2) \cot^2 \psi_s + 2(W_s^2 L^2 + M_s^2 c^2 JK) \cot \phi_s \cot \psi_s = 4W_s^2 M_s^2 c^2 (W_s^2 - M_s^2 c^2).$$

Solution of this equation for $\tan \phi_s$ and $\tan \psi_s$ in turn yields the desired equations.

### (J) Oblique Elastic Impact, Particle $n$ Initially at Rest in the Observer's Coordinate System

This corresponds to Section (D), Part I. However, here we shall derive the result by merely substituting $b=0$ in the equations of Section (I), obtaining

$$u_s = a_s \left[ \frac{mc[mc + n(c^2 - a_s^2)^{\frac{1}{2}}] \cos \phi_s \pm [m(c^2 - a_s^2)^{\frac{1}{2}} + nc](n^2 - m^2 \sin^2 \phi_s)^{\frac{1}{2}}}{m^2 c^2 + n^2 c^2 + 2mnc(c^2 - a_s^2)^{\frac{1}{2}} - m^2 a_s^2 \sin^2 \phi_s} \right]. \tag{6rd}$$

Similarly, we obtain for $v_s$,

$$v_s = \frac{2mca_s[mc + n(c^2 - a_s^2)^{\frac{1}{2}}] \cos \psi_s}{n^2(c^2 - a_s^2) + m^2 c^2 + 2mnc(c^2 - a_s^2)^{\frac{1}{2}} + m^2 a_s^2 \cos^2 \psi_s}. \tag{7rd}$$

Similarly, for the quasi velocities, we obtain

$$\eta_s = \alpha_s \left[ \frac{m[m + n(1+\alpha_s^2)^{\frac{1}{2}}] \cos \phi_s \pm [m(1+\alpha_s^2)^{\frac{1}{2}} + n](n^2 - m^2 \sin^2 \phi_s)^{\frac{1}{2}}}{m^2 + n^2 + 2mn(1+\alpha_s^2) + m^2 \alpha_s^2 \sin^2 \phi_s} \right], \tag{6r'd}$$

$$v_s = \frac{2m\alpha_s[n + m(1+\alpha_s^2)^{\frac{1}{2}}] \cos \psi_s}{m^2 + n^2 + 2mn(1+\alpha_s^2)^{\frac{1}{2}} + m^2 \alpha_s^2 \sin^2 \psi_s}. \tag{7r'd}$$

These relations are illustrated in Figs. 6 and 7.

### Collision of Energetic Electrons in the Cloud Chamber

This is the only type of relativistic collision yet attainable in the laboratory. If we let $m=n$, Eqs. (6rd), (7rd), (6r'd) and (7r'd) become

$$u_s = \frac{2a_s[c^2 + c(c^2 - a_s^2)^{\frac{1}{2}}] \cos \phi_s}{2[c^2 + c(c^2 - a_s^2)^{\frac{1}{2}}] - a_s^2 \sin^2 \phi_s}$$

together with a similar equation for $v_s$, merely changing $\psi_s$ for $\phi_s$,

$$\eta_s = \frac{2\alpha_s[1 + (1+\alpha_s^2)^{\frac{1}{2}}] \cos \phi_s}{2[1 + (1+\alpha_s^2)^{\frac{1}{2}} + \alpha_s^2 \sin^2 \phi_s},$$

and a similar expression for $v_s$. These relations are shown graphically in Fig. 8.

The relation connecting $\phi_s$ with $\psi_s$ reduces to

$$\tan\phi_s \tan\psi_s = \frac{W_s^2 - M_s^2c^2}{W_s^2} = \frac{2}{(1+\alpha_s^2)^{\frac{1}{2}}+1}.$$

Now, for two perpendicular lines, we have the well-known relation $\tan\phi \tan\psi = 1$. Remembering this, we can see from the foregoing equation that, in the collision of two electrons of equal masses, $\phi_s+\psi_s$ is always less than a right angle.

The familiar equation of the classical electrodynamics, $mu^2/\rho = Heu$, becomes in the relativistic case

$$\frac{mc}{(c^2-u_s^2)^{\frac{1}{2}}}\frac{u_s^2}{\rho_s} = H_s e u_s,$$

from which we obtain

$$\eta_s = (e/mc)H_s\rho_s = H_s\rho_s/1700.$$

Thus, we see that $\eta_s$ is the quantity actually measured by the cloud chamber when $e/m$ is known. Furthermore, since all the rest masses are assumed to be alike, it is usual to regard $\alpha_s$, $\beta_s$, $\eta_s$ and $v_s$ as quasi momentums instead of quasi velocities.

Since $\alpha_s$, $\eta_s$, $v_s$, $\phi_s$ and $\psi_s$ can be measured, in principle at least, it is possible to test these relations experimentally; or, if we assume these relations to be correct (for elastic collisions), it is possible to study the nature of the observed collision.

The only assumptions made in deriving these equations were that the total energy possessed by the colliding particles is the same after impact as before—that is, no energy, in the case of electronic collisions, is lost as radiation—and that momentum is conserved in the collision. If, now, the equations for elastic collision of like masses are not apparently obeyed, we can fall back on the more general case, and pursue one or the other of two courses: (*a*) assuming the masses to be alike, we can determine the energy loss (or gain); (*b*) assuming the collision to be elastic, we can determine the ratio of the masses of the particles in terms of the mass of the struck particle (the one initially at rest in the observer's frame). However, the data are not, as yet, accurate enough[13] to justify such a calculation.

[13] Ruark and Jones, Phys. Rev. **53**, 454 (1938).

## III. Collisions between quanta and material particles

### (K) The Photoelectric Effect

The photoelectric effect may be regarded as a two-particle inelastic impact. Before the collision the two particles are the quantum of energy $h\nu_0$ and momentum $h\nu_0/c$, and an atom or molecule of mass $n$, initially at rest with respect to the observer; after the collision, the two particles are the ejected electron and the ionized atom or molecule, which we may still think of as having mass $n$; the total momentum is still $h\nu_0/c$, but the sum of the kinetic energies of the two particles is now $h\nu_0-w$, where $w$ is the binding of the removed electron.

In the case of low energy quanta, we can substitute the foregoing quantities into Eqs. (6e) and (7e). Then, remembering that $n/m = 10^5$ for such atoms as Cu, Ni, Fe, Zn, etc., we see that the velocity and kinetic energy imparted to the residual atom are negligible, and that the kinetic energy of the electron is independent of the angle of emission, being given by $E_m = h\nu_0 - w$.

For energetic x-rays or for $\gamma$-rays, we use the corresponding relativistic equations to get the same result.

### (L) The Compton Effect

From the point of view of this article (energy-momentum relations), the Compton effect probably has received better treatment in the literature than any other type of collision. Here we have either a free electron or one whose binding energy in any molecule is negligible compared with the energy of the incident photon. Momentum is conserved by the emission of a secondary, or *scattered*, photon of less energy and momentum.

Resolving the momentums after the collision into components in the same manner as we have done before, we have

$$\begin{aligned} h\nu_0/c &= h\nu/c \cos\psi + mc\eta\cos\phi, \\ 0 &= h\nu/c \sin\psi - mc\eta\sin\phi, \end{aligned} \tag{11}$$

and for the energy equation,

$$h\nu_0 = h\nu + mc^2[\sqrt{(1+\eta^2)}-1]. \tag{21}$$

Solving this for $\eta$, we obtain

$$\eta = \frac{2(mc^2+h\nu_0)h\nu_0\cos\phi}{(mc^2+h\nu_0)^2-(h\nu_0\sin\phi)^2}. \tag{61}$$

However, it is more frequently desired to know the kinetic energy of the ejected electron than its momentum. Denoting this kinetic energy by $E_m$ as usual, we have

$$E_m = mc^2[\sqrt{(1+\eta^2)}-1],$$

or

$$mc^2\eta = \sqrt{(E_m^2+2mc^2E_m)}.$$

Substitution of this last relation in Eqs. (1l) and (2l), and solution of the resulting equations for $E_m$ gives

$$E_m = \frac{2mc^2(h\nu_0\cos\phi)^2}{(mc^2+h\nu_0)^2-(h\nu_0\cos\phi)^2} = \frac{2mc^2(h\nu_0)^2}{(mc^2+h\nu_0)^2\tan^2\phi+m^2c^4+2h\nu_0mc^2}. \quad (6l')$$

For the energy of the scattered photon, we get

$$h\nu = \frac{mc^2h\nu_0}{mc^2+h\nu_0(1-\cos\psi)}. \quad (7l)$$

Substituting these solutions back into the given Eqs. (1l), transposing and dividing one by the other, we obtain, in turn,

$$\tan\phi = \frac{mc^2\sin\psi}{(mc^2+h\nu_0)(1-\cos\psi)} = \frac{mc^2}{mc^2+h\nu_0}\cot\tfrac{1}{2}\psi,$$

$$\tan\psi = \frac{m^2c^4\sin\phi\cos\phi}{2mc^2h\nu_0+((h\nu_0)^2+m^2c^4)\sin^2\phi},$$

$$= \frac{m^2c^4\sin\phi\cos\phi}{2mc^2h\nu_0\cos^2\phi+(h\nu_0+mc^2)^2\sin^2\phi},$$

$$= \frac{m^2c^4}{2mc^2h\nu_0\cot\phi+(h\nu_0+mc^2)^2\tan\phi},$$

$$= \frac{m^2c^4\tan\phi}{2mc^2h\nu_0+(h\nu_0+mc^2)^2\tan^2\phi}. \quad (15l)$$

These relations are shown graphically in most of the books that treat the Compton effect.[14]

Equation (7l) is of especial interest. In the first place, if we consider the collinear collision, we get in the limiting case of very energetic incident photons, $\lim_{h\nu_0\to\infty} h\nu = \frac{1}{2}mc^2$ for $\psi = 180°$. Also, for any incident quantum whatever, weak or energetic, we get for the difference in wave-length between the incident and scattered quanta, $\Delta\lambda = (c/\nu)-(c/\nu_0) = h(1-\cos\psi)/mc = 2.42\times10^{-10}$ cm when $\psi = 90°$. This is equal to the wave-length of the photon whose energy is equal to the rest energy of the electron; that is, of the photon for which $h\nu = mc^2 = 0.51$ Mev. If the incident quantum has this energy ($h\nu_0 = mc^2$), we have for the electron ejected in the forward direction

$$\eta = 4/3, \qquad W_m = 5mc^2/3, \qquad E_m = 2mc^2/3 = 0.33 \text{ Mev}.$$

We now wish to use the Compton effect to measure the energy of an unknown $\gamma$-ray. Eliminating $h\nu$ from Eqs. (1l) and (2l), we have

$$h\nu_0 = \frac{mc^2[\sqrt{(1+\eta^2)}-1]}{\eta\cos\phi+\sqrt{(1+\eta^2)}-1} = \frac{mc^2[\sqrt{(1+\eta^2)}-1+\eta]}{2}, \quad (7l'')$$

when $\phi = 0$. Thus, if we can measure $\eta$ and $\phi$, we can determine $h\nu_0$. However, these recoil electrons are ejected from some thin solid suspended in the cloud chamber. It is not possible to measure the initial values of $\eta$ and $\phi$ owing to scattering as the electrons emerge from the "radiator." Statistical methods, based on the Klein-Nishina formula,[15] have been devised by Richardson and Kurie[16] to overcome this. When this method was applied to two known $\gamma$-rays, (0.51 Mev, the annihilation radiation of the positrons from $N^{13}$, and the 2.6-Mev radiation from ThC''), the results checked nicely. This is perhaps the best available experimental evidence for the validity of this treatment of the Compton effect.

In the limit of very energetic $\gamma$-rays, Eq. (7l'') reduces to

$$h\nu_0 = E_m + \tfrac{1}{2}mc^2 = E_m + 0.26 \text{ Mev},$$

when $\phi = 0$, in agreement with the limiting value of the energy for $h\nu$.

## Appendix

With reference to the expression occurring in the numerator and in the denominator of Eq. (6r'a), it is to be proved that

$$\frac{c^2-\mathbf{a}_s\cdot\mathbf{b}_s}{[(c^2-a_s^2)(c^2-b_s^2)]^{\frac{1}{2}}} = \frac{c^2-\mathbf{a}_g\cdot\mathbf{b}_g}{[(c^2-a_g^2)(c^2-b_g^2)]^{\frac{1}{2}}}.$$

From Eqs. (24), we have

$$a_{sx}b_{sx} = \frac{c^4(a_{gx}+V)(b_{gx}+V)}{(c^2+\mathbf{a}_g\cdot\mathbf{V})(c^2+\mathbf{b}_g\cdot\mathbf{V})}$$

$$a_{sy}b_{sy} = \frac{c^2a_{gy}b_{gy}(c^2-V^2)}{(c^2+\mathbf{a}_g\cdot\mathbf{V})(c^2+\mathbf{b}_g\cdot\mathbf{V})}$$

$$a_{sz}b_{sz} = \frac{c^2a_{gz}b_{gz}(c^2-V^2)}{(c^2+\mathbf{a}_g\cdot\mathbf{V})(c^2+\mathbf{b}_g\cdot\mathbf{V})}$$

$$c^2 = \frac{c^2(c^2+a_{gx}V)(c^2+b_{gx}V)}{(c^2+\mathbf{a}_g\cdot\mathbf{V})(c^2+\mathbf{b}_g\cdot\mathbf{V})}.$$

Subtraction of the last equation from the sum of the first three yields

$$c^2-\mathbf{a}_s\cdot\mathbf{b}_s = c^2(c^2-\mathbf{a}_g\cdot\mathbf{b}_g)(c^2-V^2)/(c^2+\mathbf{a}_g\cdot\mathbf{V})(c^2+\mathbf{b}_g\cdot\mathbf{V}).$$

From Eq. (25), we obtain

$$[(c^2-a_s^2)(c^2-b_s^2)]^{\frac{1}{2}} = \frac{c^2[(c^2-a_g^2)(c^2-b_g^2)]^{\frac{1}{2}}(c^2-V^2)}{(c^2+\mathbf{a}_g\cdot\mathbf{V})(c^2+\mathbf{b}_g\cdot\mathbf{V})}.$$

Division of this last equation into the one immediately preceding it gives the desired result.

Squaring both sides of Eqs. (1r) and (2r), and subtracting, we have

$$W_s^2-M_s^2c^2 = c^4\left(m^2+n^2+2mn\frac{c^2-\mathbf{a}_s\cdot\mathbf{b}_s}{[(c^2-a_s^2)(c^2-b_s^2)]^{\frac{1}{2}}}\right)$$

$$= c^4\left(m^2+n^2+2mn\frac{c^2-\mathbf{u}_s\cdot\mathbf{v}_s}{[(c^2-u_s^2)(c^2-v_s^2)]^{\frac{1}{2}}}\right).$$

Therefore, if $W_s$ is conserved, the expression in question is the same before and after the collision.

[14] For example, Ruark and Urey, *Atoms, molecules and quanta* (McGraw-Hill, 1930), pp. 86 and 87.

[15] Klein and Nishina, Zeits. f. Physik **52**, 853 (1928).

[16] Richardson and Kurie, Phys. Rev. **50**, 999 (1936).

# Best Definition with the Pinhole Camera

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IN a previous paper[1] in this journal (hereinafter referred to as I) the theory of the pinhole camera was developed. It was shown that the condition for maximum resolving power in the picture requires a pinhole appreciably larger than that found experimentally to give the best definition. The difficulty met in trying to derive theoretically the condition for best definition is that there is no obviously correct criterion for it. In this paper the question of what it is in a picture that makes for good definition is discussed further. Two possible criteria are suggested and are shown to lead to a result in satisfactory agreement with the experimental one. The optimum diameter $a$ of the pinhole is given by the expression

$$a=2[0.67\times\lambda vu/(u+v)]^{\frac{1}{2}}, \qquad (1)$$

where $\lambda$ is the wave-length of the light, $u$ is the distance from object to pinhole and $v$ is the mean distance from plate to pinhole.

The basic question is, what do we mean by good definition in a photograph? The photographic resolution of two neighboring luminous points is a problem met directly in the simple form only in astronomy where the pinhole camera is singularly useless. Ordinary objects include light and dark regions whose images are, for the most part, much larger than the distance between barely resolvable point images. The definition in such a picture probably depends more on the apparent sharpness of the boundaries between these regions than on the resolution of very fine detail. Let us make the assumption that the definition is best in the picture in which the boundaries between adjacent light and dark areas are sharpest. This seems plausible enough but constitutes no advance until we arrive at some physical measure of sharpness of such a boundary. At first glance this seems to be a simple problem, but it soon appears that the nature of the variation in the intensity in the neighborhood of the image of a boundary is different according to the shape of the image of a point source, so that there is no obvious measure of the sharpness. The matter may perhaps be made clearer by consideration of a simple special case.

Let us assume that we are dealing with the image of an area which is half white and half black with a straight line of separation between the halves. Further, assume that a picture is made of this with an ordinary camera badly out of focus so that the circle of confusion for the image of a point of the object is large compared to its diffraction pattern. The image of each point can thus be considered to be a small disk of uniform intensity having the diameter $c$. The center of the circle can be considered to be the true location of the image. In Fig. 1($a$), let the line $AB$ represent the locus of the centers of the circles of confusion corresponding to points in the object at the edge of separation between the white and black regions. To the right is the bright part of the image. The intensity at any point is to be found by drawing a circle of radius $\frac{1}{2}c$ around it and considering which points of the object give circular disks of confusion whose centers lie within the circle drawn. If their centers lie within this circle, the disks will overlap the

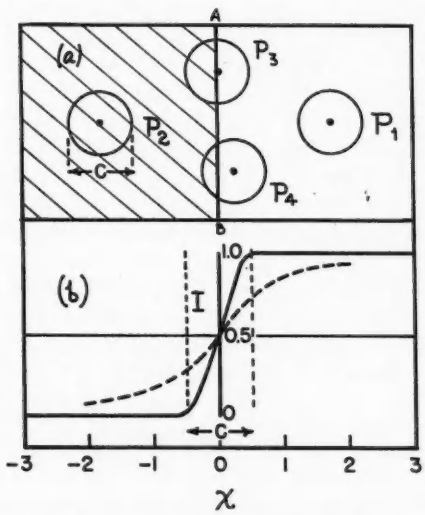


FIG. 1. Variation of intensity near an edge.

[1] Turner, Am. J. Phys. **8**, 112 (1940).

given point and contribute to the intensity of the illumination there; if they lie outside, they will add nothing to the intensity at the center of the circle. Since, in the special case under consideration, the intensity is uniform over each circular disk of confusion if it corresponds to a small bright portion of the source of fixed area and is zero if it corresponds to the dark part, the intensity of illumination at each point will be proportional to the fraction of the area of the circle of radius $\frac{1}{2}c$ surrounding it that lies to the right of $AB$. For all points such as $P_1$ at a distance greater than $\frac{1}{2}c$ from $AB$ the intensity is the same and can be taken as 1. For all points like $P_2$ more than $\frac{1}{2}c$ to the left of $AB$, it is 0, and for points on $AB$ it is $\frac{1}{2}$. For the general point like $P_4$ at a distance $x$ to the right of $AB$, $x$ being between $-\frac{1}{2}c$ and $+\frac{1}{2}c$, the intensity is given by

$$I=\frac{4}{\pi c^2}\left\{\frac{c^2}{4}\cos^{-1}\left(-\frac{2x}{c}\right)+x((\tfrac{1}{2}c)^2-x^2)^{\frac{1}{2}}\right\}. \quad (2)$$

The heavy curve of Fig. 1($b$) shows the intensity as a function of $x$. In this simple case the width of the transition region, $c$, is a convenient inverse measure of the sharpness of the edge.

Similar considerations apply when the blurring of the edge is caused by diffraction. In this case, however, there is no small disk of definite diameter $c$ over which the intensity is uniform. Suppose that a very small luminous area in the object is one which would give an image of size $\Delta A$ if the diffraction was negligible but that it actually produces a circular diffraction pattern which has an intensity $i_0$ at the center and $i(\zeta)$ at a distance $\zeta$ away from the center. If the object is a luminous area of constant intensity and of indefinite extent, the intensity at a point of its image is given by the superposition of the diffraction patterns of all the points surrounding it. Those lying at distances between $\zeta$ and $\zeta+\mathrm{d}\zeta$ will occupy an area of $2\pi\zeta\mathrm{d}\zeta$ and contribute an amount $(2\pi\zeta\mathrm{d}\zeta/\Delta A)i(\zeta)$ to the intensity. The total intensity $I_t$ will be given by

$$I_t=(2\pi/\Delta A)\int_0^\infty \zeta i(\zeta)\mathrm{d}\zeta. \quad (3)$$

For a point at a distance $x$ away from an infinite edge such as that of Fig. 1 the corresponding expressions are

$$I_{x<0}=\frac{2\pi}{\Delta A}\int_{-x}^{\infty}\frac{\cos^{-1}(-x/\zeta)}{\pi}\zeta i(\zeta)\mathrm{d}\zeta, \quad (4a)$$

$$I_{x>0}=\frac{2\pi}{\Delta A}\left[\int_0^{+x}\zeta i(\zeta)\mathrm{d}\zeta+\int_x^{\infty}\frac{\cos^{-1}(-x/\zeta)}{\pi}\zeta i(\zeta)\mathrm{d}\zeta\right]. \quad (4b)$$

The new factor in Eqs. (4$a$) and (4$b$), $\cos^{-1}(-x/\zeta)/\pi$, gives the fraction of the ring of radius $\zeta$ to $\zeta+\mathrm{d}\zeta$ that contains luminous centers. The upper limit of $\infty$ is allowable merely because the intensity does drop off rapidly with increasing $\zeta$, so that those centers which are remote contribute a negligible amount to the diffraction pattern. The presence of the factor $\zeta$ in the integral, however, gives increased relative importance to the outer parts of the diffraction pattern as compared with the argument involved in finding ordinary resolving power.

There are certain important properties of the integrals in Eqs. (4$a$) and (4$b$) that may be seen without actually evaluating them for the different diffraction patterns. For large positive values of $x$ the intensity approaches $I_t$, the constant value given by the integral (3), in agreement with the common-sense aspect of the matter that the intensity in the image of the bright area at some distance away from the edge is practically unaffected by the presence of the edge and dark region. Similarly, for large negative values of $x$ the intensity approaches zero. For $x=0$, $\cos^{-1}(-x/\zeta)/\pi$ becomes $\frac{1}{2}$ and the intensity is $\frac{1}{2}$ the full value in the light area. Thus, the curve of intensity as a function of $x$ will, in a general way be like the solid curve of Fig. 1($b$), except that the limiting values of $I_t$ and 0 will not be reached anywhere. From the geometrical symmetry, it is apparent that $I(-x)=I_t-I(x)$. The curve will thus exhibit the same symmetry with respect to a rotation of 180° about the midpoint as that shown by the solid curve of Fig. 1($b$). The midpoint is a point of inflection which means that the slope remains fairly constant over a considerable range of $x$ in the neighborhood of $x=0$. It is obvious that as the diffraction pattern of a point becomes more spread out this curve of $I$ *versus* $x$ will become flatter with a smaller value of the

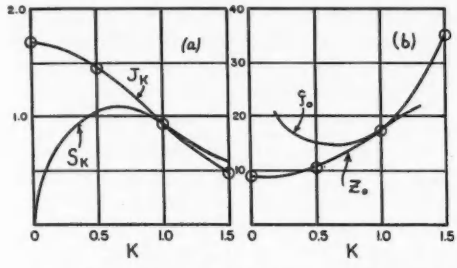


FIG. 2. Determination of the optimum size of the pinhole: (*a*) for maximum sharpness of edges, (*b*) for minimum size of perceptible black disks.

slope at the center. There is no limited region of transition from full intensity to zero as with the out-of-focus image, and therefore no related measure of the lack of sharpness of the edge. The sharpness may be measured, however, by the slope of the central portion of the curve, nearly equal to $(dI/dx)_0$ over a considerable range of $x$. To calculate the integrals (4*a*) and (4*b*) for different functions $i(\zeta)$ corresponding to pinholes of different size and for different values of $x$ is laborious. The dashed curve of Fig. 1(*b*) is one such computed curve. Fortunately, it is not necessary to do this for all values of $y$ in order to get accurate values of $(dI/dx)_0$. Differentiation of Eq. (4a) with respect to $x$, followed by setting $x$ equal to zero, gives

$$(dI/dx)_0=(2/\Delta A)\int_0^\infty i(\zeta)d\zeta. \qquad (5)$$

If the intensity is to be referred to its maximum value taken as 1, Eq. (5) must be divided by Eq. (3) to get

$$(dI_{rel}/dx)_0=\int_0^\infty i(\zeta)d\zeta \Big/ \int_0^\infty \zeta i(\zeta)d\zeta. \qquad (6)$$

The calculation of the values of $(dI_{rel}/dx)_0$ for pinholes of different sizes is based on the work of Lommel, discussed in I. That discussion and the definition of Lommel's variables will not be repeated here. Introducing Lommel's $z$ and $y$, and replacing $i(\zeta)$ by $M^2(z)$, to which it is proportional, Eq. (6) becomes

$$(dI_{rel}/dx)_0=y^{\frac{1}{2}}\int_0^\infty M^2(z)dz \Big/ k\int_0^\infty M^2(z)\cdot z dz. \qquad (7)$$

Eq. (3) of I has been taken as $\zeta=kz/y^{\frac{1}{2}}$, $k$ being a constant which is independent of the size of the pinhole. The integral in the denominator,

$$\int_0^\infty M^2(z)\cdot z dz,$$

is found to have the value of 2 for all values of $y$, as will be shown in the Appendix. If $y$ is expressed as $K\times 2\pi$, as in I, the sharpness for each size of pinhole is proportional to

$$S_K=\frac{2k(dI_{rel}/dx_0)}{(2\pi)^{\frac{1}{2}}}=K^{\frac{1}{2}}\int_0^\infty M_K{}^2(z)dz.$$

Approximate values of the integrals,

$$J_K=\int_0^\infty M_K{}^2(z)dz,$$

may be obtained by graphical integration, using Lommel's tables in which he gives $M^2(z)$ for integral values of $z$ up to 12 for values of $y$ corresponding to $K=0$, $\frac{1}{2}$, 1, $\frac{3}{2}$, etc. In Fig. 2(*a*) the computed values of these integrals have been plotted as a function of $K$ and a curve drawn through the computed points. Multiplication of the ordinates of this curve by the respective values of $K^{\frac{1}{2}}$ gives the curve for $S_K$ which is also drawn. It is seen to have a maximum at $K=0.67$, corresponding to $y=1.34\pi$. The location of this maximum is not very sensitive to changes in the way in which the $J_K$ curve is drawn through the four computed points. All the values for the ordinates of these points are slightly too low, since the integrations are extended only to $z=12$, instead of to $z=\infty$. The errors are negligible, however, because of the very rapid decrease of $M^2(z)$ with increasing $z$. Inserting the value for $K_{max}$ into Eq. (1) of I gives the expression for $a$, the diameter of the pinhole for sharpest edges, as

$$a=2[0.67\lambda vu/(u+v)]^{\frac{1}{2}},$$

the result already given in the first paragraph. That $v$ is to be taken as the mean distance rather than the perpendicular distance will be explained later.

Before comparing this result with Lord Rayleigh's experimental one, another interpretation of its meaning should be mentioned. Consider the image of a black strip of constant width crossing a uniformly illuminated white area. The

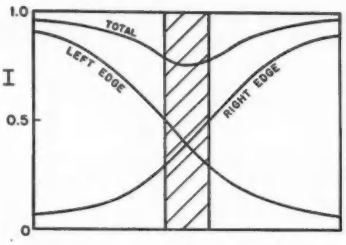


FIG. 3. Intensity in the image of a black strip.

intensity at any point in the region of the perfect geometric image of the black strip will be the sum of the two intensities which would exist at the point if the black area should be extended indefinitely in either the one or the other direction. To express it otherwise, the intensity curves from the two edges overlap, giving a total intensity that is the sum of the two separate ones. A possible case is illustrated in Fig. 3. The two vertical lines mark the limits of the edge of the geometric image of the black strip. The two separate intensity curves for the edges are drawn in and also the curve for the total intensity which has a minimum at the center. In spectroscopy, lines are found to be resolvable when such a minimum has an intensity of 0.8 or less. If we apply this result here, it means that the black strip will not be seen unless its width is as great as or greater than twice the distance in which the intensity at a single edge drops from 0.5 to 0.4, so that the superposed intensity at the center will be less than 0.8. This distance, however, varies inversely as $(dI/dx)_0$. In the foregoing discussion we have found the condition for obtaining the maximum possible value of $(dI/dx)_0$. This is now seen to be also the condition for having minimum width of a just perceptible black strip in the object. It is also apparent that the result for the optimum size of pinhole does not depend on the particular choice of 0.8 for the intensity at the minimum. The same result will be obtained for any value which is close enough to 1 so that a range of the curve of the single edge is used over which $dI/dx$ is practically constant and equal to $(dI/dx)_0$.

This raises the question whether the particular result obtained is valid only for obtaining minimum size of long strips. What about the size of just perceptible dark squares or circles? A simple solution can be had for the circles. By the argument which leads to Eq. (3), it is apparent that the intensity at the center of the image of a black circular dot of radius $\zeta_0$ is given by

$$I_d=(2\pi/\Delta A)\int_{\zeta_0}^{\infty}\zeta i(\zeta)d\zeta. \tag{8}$$

For the dot to be perceptible $I_d$ must be equal to or less than 0.8 of $I_t$. The radius $\zeta_0$ of the smallest perceptible dot is given by

$$\int_{\zeta_0}^{\infty}\zeta i(\zeta)d\zeta=0.8\int_0^{\infty}\zeta i(\zeta)d\zeta$$

or

$$\int_0^{\zeta_0}\zeta i(\zeta)d\zeta=0.2\int_0^{\infty}\zeta i(\zeta)d\zeta.$$

Changing variables, this gives

$$\int_0^{z_0}zM^2(z)dz=0.2\int_0^{\infty}zM^2(z)dz.$$

Since the integral on the right has the value of 2 (see Appendix), one has

$$\int_0^{z_0}zM^2(z)dz=0.4. \tag{9}$$

In Fig. 4, $zM^2(z)$ is plotted as a function of $z$ for $y=2\pi$. It should be noticed how important the second maximum at $z=3.8$ has become as compared with the curve of $M^2(z)$ (intensity) as a function of $z$ (Fig. 1 of I). The integral of the left-hand member of Eq. (9) has been found for various assumed values of $z_0$ by use of a planimeter and the value of $z_0$ to give 0.4 for the integral obtained by interpolation. The results of such calculations for several different values of $y$ (and $K$) are plotted in Fig. 2(*b*), and a curve drawn through them. This is now to be converted into a curve for $\zeta_0$, or rather a quantity proportional to it, by dividing each ordinate by $K^{\frac{1}{3}}$, since $\zeta=kz/y^{\frac{1}{3}}$ and $y=K\times 2\pi$. The resulting curve, labeled $\zeta_0$ in Fig. 2(*b*), shows that the just perceptible circular dot will be of smallest size for $K=0.67$, in agreement with the value first obtained. This second result also is very insensitive to the particular value taken for the critical intensity at the center of the image of the circular dot (here taken to be 0.8).

The size for the pinhole here obtained is thus one that gives maximum sharpness of boundaries between light and dark regions and also is one for which the size of just perceptible dark detail is smallest, but is quite different from the one for maximum resolving power of adjacent points. In the foregoing argument the intensity of light coming from the dark regions has been taken to be zero, for simplicity's sake. The same results for the sharpness of edges will be obtained for any assumed value of the lower intensity. However, the argument about the minimum width of a perceptible darker region will become more complicated if, with increasing brightness, its width becomes so great that at the center of its image the slope of the overlapping $I$ *versus* $x$ curves (Fig. 3) departs greatly from $(\mathrm{d}I/\mathrm{d}x)_0$.

It may be objected that proper account has not been taken of the fact that it is the logarithm of the intensity rather than the intensity itself which is important both psychologically and photographically. One might think that the maximum value of $\mathrm{d}\log I/\mathrm{d}x$ would be the proper measure of the sharpness of an edge rather than the maximum value of $\mathrm{d}I/\mathrm{d}x$. One runs into difficulties in trying to apply this criterion. For the blurred edges arising from imperfection of focus $\mathrm{d}\log I/\mathrm{d}x$ becomes infinite where the intensity goes to zero, irrespective of the size of the circle of confusion, so that in this case it is not a useful measure of sharpness. Perhaps an average value of $\mathrm{d}\log I/\mathrm{d}x$ over some standard range of intensity would be the correct measure. Here one encounters the difficulty concerning arbitrary choice of the appropriate range of intensity. The method followed in the foregoing argument seemed to be the only reasonably straightforward one not involving such arbitrary assumptions.

The new theoretical result for the optimum size of pinhole, $y=1.34\pi$, is in apparent disagreement with the experimental results of Lord Rayleigh (see I) who found $1.8\pi$. This discrepancy can be attributed to the size and the nature of the test objects which he used. One of these was a pair of equal round holes of unspecified size $\frac{1}{4}$ in. apart in a piece of metal; the other was a grating cut from sheet zinc with a grating space of $\frac{1}{4}$ in. They were set up at a distance of 18 ft from the pinholes and illuminated from behind. The image was viewed or photographed at a distance of 8 in. behind the pinholes. The geometric image of a single element of the grating would have had a width of $(0.25\times8)/(12\times18)=0.0092$ in. By means of Eq. (3) of I, one can compute that this corresponds to a $z$ value of 3.9, if $y$ be taken as $1.8\pi$. This is small enough so that the diffracted images of the successive openings of the grating overlapped considerably. The apparent sharpness at any edge depended on the presence of the others in its neighborhood, so the arrangement was much more complicated than that considered in the theory. For best definition of exceedingly fine spread-out structure, undoubtedly the condition for maximum resolving power is the relevant one. Rayleigh's experimental grating was an object having a regularity and fineness of detail not encountered in the objects ordinarily photographed. It should be noticed that the size of the image of one grating space, 0.0092 in., is only slightly greater than the 0.008 in. which is often taken to be the upper tolerable limit of the circle of confusion for good focus with a camera having a focal length of 8 in. Since Rayleight does not give the sizes of the two holes in the plate which constituted the other test object, little can be said about his results with it. If the holes were small compared to the distance between them, then the object approximated two point sources and the condition for maximum resolving power, $y=2.4\pi$, would have been expected to be best. For holes of size comparable with the distance between their centers, something more like the result found here, $1.34\pi$, would have given the best result. It is perhaps significant that Rayleigh did find that the best size of pinhole was slightly smaller for this object than it was for his grating. Ap-

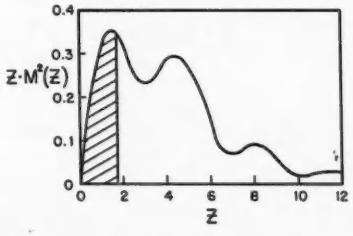


FIG. 4. Graphical method of determining the size of the smallest perceptible black disk. Shaded area=0.4.

parently there is no size of hole that is best for all objects. The one corresponding to $y=1.34\pi$ is merely the one that is best for the less exacting conditions of ordinary photography.

It is now possible to compare the definition obtainable with the optimum size of pinhole with that obtained with an ordinary camera, in a more precise way. We may find the size of the circle of confusion which will give the same sharpness of edge as measured by $(dI/dx)_0$ as does the optimum pinhole. By differentiating Eq. (2), one gets $(dI/dx)_0=4/\pi c$ for the diffuse edge of the out-of-focus camera. This is set equal to $(dI/dx)_0$ from Eq. (7), where $y=1.34\pi$.

$$\int_0^\infty M^2(z)dz=1.32 \quad \text{for} \quad y=1.34\pi \text{ [Fig. 2(a)]},$$

$$\int_0^\infty zM^2(z)dz=2 \text{ (Appendix)},$$

and $k=[\lambda v(1+v/u)/2\pi]^{\frac{1}{2}}$. Let $u\gg v$ and take $\lambda=4.5\times10^{-5}$ cm; then

$$1.32[1.34\pi]^{\frac{1}{2}}/2[4.5\times10^{-5}v/2\pi]^{\frac{1}{2}}=4/\pi c.$$

This gives $v/c=396\,v^{\frac{1}{2}}$. For $v=9$ cm, $v/c=1190$. Since $v/c\geqslant1000$ is often taken as the criterion for satisfactory focus with ordinary cameras, this means that pinhole cameras of focal length of 9 cm or more should give pictures comparable with those of ordinary cameras if the optimum size of pinhole is used. This neglects the blurring that results because the diffraction patterns produced by light of different wave-lengths are of different size.

The foregoing theory is strictly valid only for regions very close to the axis of the pinhole that is normal to the plate. The diffraction patterns of oblique beams will be somewhat modified, and it might be feared that this would alter the definition seriously at the margin of the picture. Such an effect is not a prominent one experimentally and it may readily be seen, as follows, to be of slight importance under ordinary circumstances. For a ray passing through the hole making an angle $\theta$ with the axis, the diffraction pattern will be elongated radially, partly because of its oblique incidence on the plate, but also because the pinhole is effectively an elliptical aperture. Without going into details of the diffraction by elliptical holes, one can see that the pattern will be elongated in a direction at right angles to the long axis of the ellipse (that is, radially) in the same way that the pattern from a rectangular aperture is similar to the aperture but rotated by 90°. To a first approximation this elongation would be by a factor of sec $\theta$, where $\theta$ is the angle between the axis and the bundle. The oblique view of the object (assuming it to be something like a drawing in one plane) would give another factor of sec $\theta$. For $\theta=30°$, $\sec^2\theta=1.32$. Thus, the minimum size of perceptible tangential lines should increase by a factor of 4/3 out toward the edge of a plate where the angle is 30°. The definition of radial lines should not be thus changed, so that the effect is like that of astigmatism of a lens. If we take $\theta=20°$ as a reasonable average value, $\sec^2\theta=1.15$, and the effect is seen to be very small for a considerable angle of view. The quantity $v$ of Eq. (1) should also be taken as the average hole-to-plate distance instead of the minimum distance for the best average definition over the whole plate.

## Appendix

That $\int_0^\infty zM^2(z)dz=2$ may be shown as follows. It is shown in all textbooks of optics where Kirchhoff's theory of Huygens' principle is developed that, if $A$ is the amplitude of a wave at an element in the opening in a diffracting screen, the amplitude at a point on the screen will be

$$\frac{A}{\lambda v}\int \sin 2\pi\left(\frac{t}{T}-\frac{u+v}{\lambda}\right)d\sigma,$$

the integration being taken over the opening; $u$ and $v$ in the integral are the distances from a point source to $d\sigma$ and from $d\sigma$ to the point on the screen for which the intensity is being computed, respectively. Lommel, apparently because of interest only in relative intensities, omits the $1/\lambda$ factor. He then develops this integral in the customary way to transform this expression to get the amplitude equal to $A\pi r^2M/\lambda v$, where $r$ is the radius of the hole and $M^2$ is $(C^2+S^2)\pi^2r^4$, $C$ and $S$ being the familiar diffraction integrals. Lommel calculates and tabulates the values of $M^2$ for different sizes of diffracting hole, for different radii in the pattern. The intensity at a point will be proportional to the square of the amplitude so

$$i(\zeta)=BA^2\pi^2r^4M^2(\zeta)/\lambda^2v^2,$$

$B$ being the constant of proportionality. The total energy per unit time coming to the pattern will be given by integration over the whole pattern, or

$$E=(BA^2\pi^2r^4/\lambda^2v^2)\int_0^\infty 2\pi\zeta M^2(\zeta)d\zeta.$$

This must also equal the energy entering the hole per unit time, which is $BA^2\pi r^2$. Equating these two expressions for the energy, one obtains

$$\int_0^\infty \zeta M^2(\zeta)d\zeta = \lambda^2 v^2/2\pi^2 r^2 = 2(\lambda v/2\pi r)^2.$$

A change of variable from $\zeta$ to $z$ gives, since $\zeta = (\lambda v/2\pi r)z$,

$$(\lambda v/2\pi r)^2 \int_0^\infty z M^2(z)dz = 2(\lambda v/2\pi r)^2$$

or

$$\int_0^\infty z M^2(z)dz = 2,$$

which was to be proved.

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## The Teaching of Electrostatics in the General Course

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THE writer returned this spring, after a lapse of nearly ten years, to the teaching of that part of *general* physics which includes electricity. Both he and his students have survived the experience but he, at least, is uncomfortably aware that the presentation lacked coherence and he is inclined to question whether this lack of unity is not inherent in the textbooks. Mechanics, sound and heat form a reasonably consistent whole in spite of the burden of several sets of units when one would suffice. It is probable that this consistency might be made even more evident were the authors to employ the kinetic point of view more wholeheartedly in heat, but they do not encourage the reader to regard these parts as three isolated sections. Electricity suffers from a still greater superfluity of units, and one may suspect that the new mks system, admirable as it is, will make the present confusion even worse confounded before the situation clears. But though the existence of electrostatic units and practical units is at worst an inconvenience, to suggest, if only by indirection, that there are electrostatic principles and practical principles is fatal to a true grasp of the subject.

The writer is not suggesting that the discussion of electrostatics in the books on general physics is purposely divorced from practical considerations but he is convinced that many an "intelligent reader" carries away that impression. The following is a list of experimental matters to which the laws of electrostatics might be applied in a book of this sort:

Van de Graaff generator
Millikan's determination of $e$
Cathode-ray oscillograph with electric deflection
Triode
X-ray tube
Emission of electrons from hot bodies
Ionization by impact
Scattering of $\alpha$-particles
Polarization of insulating mediums
Condensers

Many of these problems involve moving charges, of course, but the charges move primarily under electric forces, the accompanying magnetic forces being negligible.

It is evident that the dielectric constant is a useless concept in the large majority of these cases, but the reader has been led to expect otherwise. He first meets the Coulomb law in the form $F = Cq_1q_2/r^2$ as a generalization from Coulomb's experiments and then learns that the true form of the law should be $F = Cq_1q_2/kr^2$ when the charges are immersed in a ponderable medium. He is usually left to infer that the true form was discovered by carrying out the experiment with the spheres immersed in something like transformer oil—or even glass since the values of $k$ for various liquids and solids are often appended to the discussion! Occasionally an author is more candid and admits that the constant has been introduced into the law so that it may be consistent with later developments of the subject but he fails to specify the developments to which he refers.

From the experimental point of view the most precise direct experiments with point charges are those of the single scattering of $\alpha$-particles by heavy nuclei and these experiments offer no compelling reason for changing the form of the law. It is even immaterial whether the scattering medium is a conductor or an insulator. The "later developments" are more probably to be found in the growth of electric theory, specifically

in the Faraday-Maxwell distinction between $E$ and $D$. This holy of holies, the writer believes, might well remain veiled from the neophyte until he is prepared by long meditation in the lesser courts of the temple. This conclusion is based on two considerations: first, the distinction is hazy and uncertain with even the best students in an elementary section; second, the first presentation of electricity involves so few concrete situations where the distinction is of importance.

This paper, therefore, is an attempt to explore the thesis that electrostatics should be organized and developed with the practical applications clearly in mind. It is felt that this procedure may be of considerable advantage to the beginner and that it need do no violence to the more advanced theory which he may one day peruse.

It is proposed to start the discussion with a brief description of the elementary electric entities and of the electric structure of atoms. Both relative and absolute values for their masses and charges should be given. The experimental evidence for their existence and evaluation may be postponed, but the concepts should be definite and numerical from the beginning. It is all very well to state that ordinary matter contains positive and negative electricity in equal amounts and that the charging of a body merely involves disturbing this balance. But it is much more impressive to be able to show that, if the lack of balance were as much as one part in $10^{10}$ in a gram of an average metal, this gram would have to be made into a hollow sphere of at least 10-cm radius before it could retain this charge if surrounded by air.

This calculation assumes that the atomic weight is, on the average, twice the atomic number. One gram will contain $3\times10^{23}$ electrons, and the two charges will be $5\times10^{4}$ coul each. The rest of the computation follows from the condition that the field at the surface of the sphere shall not exceed 100 esu. Incidentally the charges are proportionally much less out of balance than the sexes would be in our world if there were one more male than female!

The Coulomb law should be introduced in the form

$$F = Cq_1q_2/r^2, \qquad (1)$$

where $C$ is a constant whose value depends only on the system of units employed. It is important that the experimental restrictions receive due emphasis, namely, that imposed by "point" charges and that imposed by the lack of medium *and* boundaries. It should be pointed out that the law also holds for point charges in a gaseous medium, since a gas is a fair approximation to a vacuum electrically, but no reference need be made to other mediums. The real importance of Eq. (1) lies in its application to specific experimental situations rather than to ingenious algebraic or geometric problems. If the usual lecture demonstration is carried out showing the electric repulsion of two light balls, a numerical estimate of the charge of a ball should be made and this should be compared with the total electron content and with the charge which would result if each surface atom had gained one electron. It is also worth while to consider in this connection the case of an $\alpha$-particle which has approached within $10^{-11}$ cm of a heavy nucleus where the force becomes of the order of 1 kg on a particle having a mass of about $10^{-23}$ gm. It may be pointed out that the gravitational attraction of the two at this distance is about $10^{-30}$ dyne.

The concepts of the electric field and the vector $E$ follow naturally from the Coulomb law. The law makes possible a direct computation of $E$ for specific distributions of point charges, but it must be admitted that such calculations are not often met in practice and that the mathematical manipulations may take first place in the mind of the student. It is true that, in computing such fields, he is compelled to recognize the vector aspect of $E$ but, by and large, the field of a single charge is rather more important to him than that due to three charges located at the vertices of a triangle. From the standpoint of a beginner, however, the quantitative description of field in terms of lines of force is more valuable than any number of equations giving the field due to point charges, particularly if the idea is carried through to the Gauss principle or its equivalent. For in most applications it is the electric field at or near the surface of a charged conductor that is of interest; hence, it becomes important to develop the relation between this field and the surface density of the charge.

Corona and spark discharges in gases afford an interesting example of the effects of electric fields. The explanation of the phenomenon in terms of ionization by impact combines rather

nicely certain aspects of mechanics, electrostatics, kinetic theory and molecular structure. The theory predicts both the existence of a critical field and also its variation with pressure. The variation may be demonstrated for pressures above as well as below normal without elaborate apparatus. The evaluation of the critical field is another matter but there is a simple experiment that gives its approximate value in normal air. If two ping-pong balls coated with Aquadag are suspended from a common point with threads about 1 m long, no amount of charging will separate them more than something between 10 and 20 cm. The charge corresponding to this maximum separation may be computed in the usual way. The field at the surface of one of the balls may then be found from the surface density of the charge, the result being 100 esu, more or less.

The ground is now laid for discussing the Van de Graaff generator in some detail. The mechanism for charging the belt at the lower end and for removing the charge at the upper end is a direct illustration of the idea of critical fields. A numerical estimate may also be made of the maximum rate at which a belt of given width and linear speed may carry charge; that is, the charging current may be found in amperes. Similarly, the limiting charge on the sphere may be approximated. Only the maximum potential of the sphere and the electric power of the generator need be reserved for later comment.

The discussion of electrostatic principles, as far as the needs of an introductory course are concerned, may be concluded by developing the concept of potential. The required relations are the expressions for the potential due to one or more point charges and for the potential difference between two points in a field which is either uniform or radial. The interrelation of intensity and potential is also needed. In this connection it may be remarked that potential differences in practice are measured in volts and it is essential that the definition of the volt in terms of the joule and the coulomb be incorporated in the presentation of electrostatics.

Among the possible applications of these principles the first should be the Millikan oil drop experiment, since the value and the uniqueness of the electron charge lie at the heart of electric theory. The experiment involves not only the force on a drop in terms of field and charge but also the evaluation of the field from potential difference and plate separation. An examination of six books on general physics which have appeared in the last three years shows a wide variation in the treatment of this subject. One book omits the experiment entirely, two give a brief qualitative description, two add to a qualitative account the equation for a drop just held from falling, and the sixth devotes an entire chapter to a detailed resumé of Millikan's work, including typical observations and results. It may be argued that the authors of the last book have overemphasized the subject, but their perspective is surely more nearly right than that of an author who dismisses it with a page. Incidentally the apparatus may well be given to the student to play with, for even though no measurements are made his imagination is caught and his insight deepened by finding himself able to control at will the motion of a drop.

The acceleration of charged particles between electrodes at fixed potentials involves a direct application of theory and should be worked through with numerical detail. There is profit in comparing the final energies and speeds of electrons and light nuclei and in setting them against the corresponding values for particles in temperature equilibrium. The cathode-ray oscillograph with electric deflection affords an opportunity for a more extended application of the same ideas. Needless to say, the marked usefulness of this device for lecture purposes justifies the effort expended in explaining its action.

The emission of electricity from hot bodies is a similar problem in that any elementary discussion of the matter centers about questions of potential and field, and there seems no good reason for relegating the effect to one of the postscript chapters. It is true, of course, that similar considerations apply to the triode, and yet one may well omit it at this point because its uses involve other principles than those of electrostatics. But the triode with voltages applied as they are in the ionization gauge makes an entertaining illustration of potential theory and may be discussed with profit.

There remains the question whether one may

omit the dielectric constant entirely from the discussion of electricity in an elementary textbook without doing violence to the logical development. As a practical matter the determination of this quantity results from measurements made with condensers and one should not confine his discussion of condensers to the case where the medium is a vacuum. The writer favors using the concept in this connection but he puts forward the following analysis of the parallel plate condenser as an indication that the dielectric constant is not a *sine qua non*.

The conventional electrostatic units are used but the train of the argument will apply in any system. The symbols carry the usual meanings. From the Gauss law the field for a vacuum between the plates is $E=4\pi Q/A$ and the resulting potential difference is $V=4\pi Qt/A$. The capacitance becomes $C_0=A/4\pi t$, where the subscript indicates the absence of a ponderable insulator. If a polarizable medium is introduced between the plates, the shift of the charges in the medium reduces the effective charge—the charge from which the field is to be calculated—from $Q$ to $Q-q$. If we make the reasonable assumption that $q=\alpha Q$, where $\alpha<1$, the field becomes

$$E=4\pi Q(1-\alpha)/A$$

and the potential,

$$V=4\pi Qt(1-\alpha)/A.$$

Then the capacitance has the value

$$C=A/4\pi t(1-\alpha)=C_0/(1-\alpha).$$

This may be written in the more usual form, $C=kC_0$, where $k>1$ since $\alpha<1$.

The foregoing analysis does no violence to the electrostatic principles; it has the advantage of calling specific attention to the mechanism of polarization, and it is consistent with the experimental fact that the capacitance is larger with an insulating medium than with a vacuum.

Before summarily rejecting this point of view, it may be well to ponder the case of the permanently polarized dielectric, the electret. To avoid needless complications, consider an infinite sheet of such a material polarized perpendicularly to the sheet. In this case $D$ vanishes at all points while $E$ vanishes at points outside the electret but not within. Hence, if we define the dielectric constant as the ratio $D/E$, we conclude that it is zero for the material of the electret, a result which is not particularly enlightening. On the other hand, the question involves no inherent difficulties if one grants the existence of actual surface charges in a polarized insulator. Since the polarization is uniform and perpendicular to the surface, $P$ equals $\sigma$ for the electret. The internal electric field is given by $E=-4\pi\sigma$, and the displacement becomes

$$D=E+4\pi P=-4\pi\sigma+4\pi P=0.$$

It is the circumstance that $P$ is not proportional to $E$ in the usual sense which makes the description of the phenomenon in terms of a dielectric constant meaningless.

It is perhaps worth noting that the discussion of the polarization of dielectrics may well serve as an introduction to electrolysis. For instance, two conducting plates separated by a medium made up of sodium and chlorine ions in equal numbers may constitute a condenser or an electrolytic cell. If the medium is in the solid state (a slab of crystal), the ions are bound, whereas in the molten state they are free to move. In each case increasing the potential difference between the plates increases the strength of the field in which the ions are located. This results in the first case in a larger displacement of the ions and a corresponding increase in polarization; in the second, it causes an increase in the speed of the ions and hence a larger current.

In conclusion, the writer frankly admits that he has outlined the way in which he would like to teach electrostatics and that his performance in the classroom fell far short of this ideal. He writes in the hope that before he is again called upon to teach the subject some author will have brought out a book on general physics in which electrostatics is discussed somewhat along these lines. He acknowledges his debt to his colleagues, M. W. Garrett and W. C. Elmore, whose comments and advice have made this paper possible.

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*NOTHING tends so much to the advancement of knowledge, as the application of a new instrument. The native intellectual powers of men in different times are not so much the causes of the different successes of their labours, as the peculiar nature of the means and artificial resources in their possession.*—HUMPHRY DAVY, *Collected Works* (1849), vol. IV, p. 37.

# The Physics Laboratory at the University of Minnesota

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AFTER one has built a home for himself, and has lived in it for a few years, he is often heard to say: "If I build again I shall (or shall not) do this or that. . . ." The editor of this journal has suggested that a physics department which has lived in a building for a decade may have more pertinent information to give on building details than can be given by a department which has just moved into a new structure. I shall, therefore, describe some of the features of the laboratory and details of equipment at the University of Minnesota that may be of interest to others in designing a new building or remodeling an old one. The details of dimensions, exterior finish and construction materials will not be described except as they are given by photographs (Fig. 1) and the floor plans (Fig. 2) which are drawn to scale. The building is adequate to serve the needs of 1000 undergraduate students, taught mainly in large lecture sections, and 25 graduate students, who for the most part will be engaged in experimental research. A staff of 9 full-time members is also housed satisfactorily.

## Lecture rooms

There are three lecture rooms, all placed on the first floor in order that the large student body will not find it necessary to go to parts of the building given over to laboratory work. The seating capacities are 500, 200 and 165. The three rooms are served by a common apparatus and preparation room (Fig. 2, first floor plan). This preparation room has no outside windows or doors through which dust may enter. In order to circulate as little dust as possible, ventilation of the room is obtained entirely through the doors to the adjacent lecture rooms. The apparatus room is equipped with cases, each shelf of which is lighted by properly placed lamps.

The lecture rooms have fixed lecture tables. We sometimes wish these were movable in sections which could be prepared with apparatus in the adjacent storeroom and rolled into the lecture room during the intermission between classes. The black tops of the tables have recently been covered with white linoleum of ordinary floor grade. This covering gives a better background for showing apparatus. The tables are equipped with multiple gas, air and electric outlets, *distributed along the table.* Heavy current, 110–220-v d.c., a.c., single-phase and three-phase electric outlets are available. There is also a large volume compressed air outlet in addition to the usual small laboratory table outlets. The lights of the

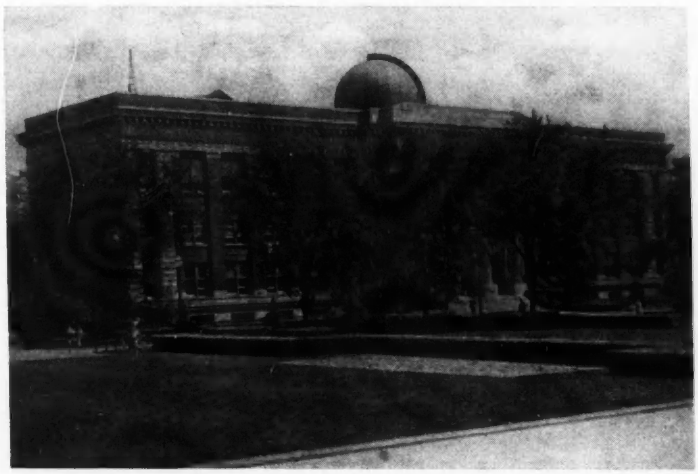
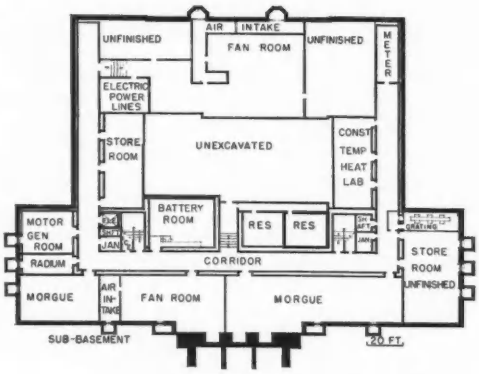
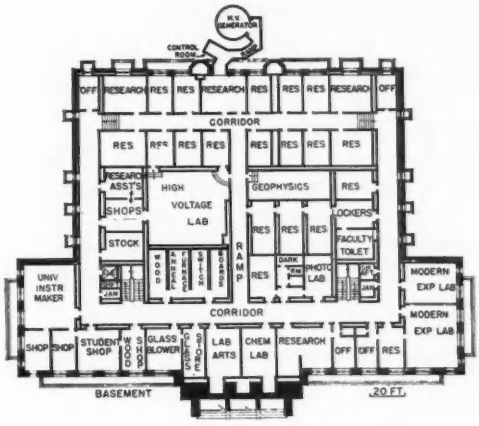


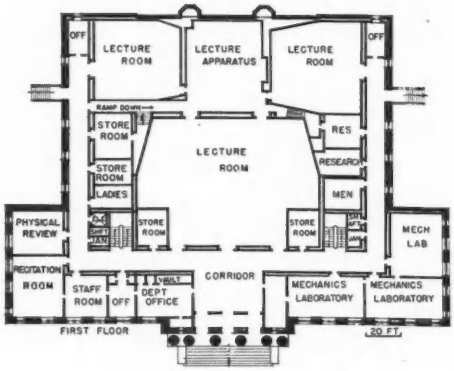
Fig. 1. Front of building.



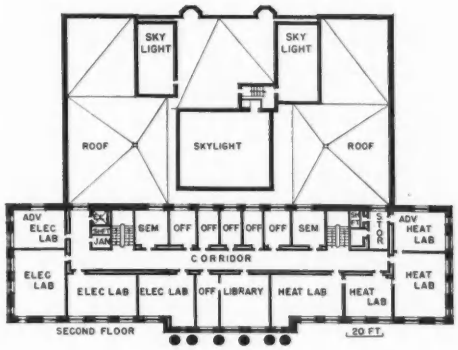

(a) Sub-basement



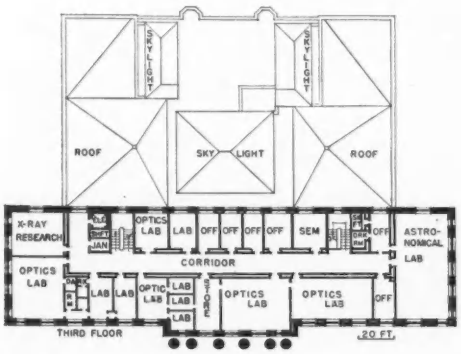

(b) Basement




(c) First floor




(d) Second floor




(e) Third floor

FIG. 2. Floor plans. Abbreviations: *Off*, office; *Sem*, seminar rooms; *Res*, research rooms; *Shaft*, vertical shaft.

rooms may be controlled from either end of the tables as well as at the entrance to the rooms. The skylights are also controlled by switches at the tables.

Projection screens of white plaster are placed on the front walls above the blackboards and on each of the side walls near the front of the rooms. These side walls are set at angles (Fig. 2, first floor), and the screens on them are used more often than those on the front wall. The lecturer himself can operate the lantern from the lecture table. Unless the audience is very large and the seats along the wall are occupied, everyone in the room can see the projections very well. These screens also serve for shadow projection of small pieces of apparatus on the lecture table. By such projection a large class can, for example, see the motions of the leaf of an electroscope.

Galvanometers are placed on the rear wall and a spot projected on the front wall. Several galvanometer outlets are situated along the lecture table and a control box which permits variation of sensitivity is placed in a drawer.

A sink and hot and cold water outlets are available at one end of the lecture table. The terrazzo floor at that end of the table has a drain and no harm is done if, in an experiment, water is spilled over the floor.

The blackboards are illuminated by lights in a trough placed above and a short distance in front of them, and by lights placed below the edge of the lecture table and directed toward the boards. The blackboards are single slate slabs. We sometimes wish they were the multiple sliding type.

The top of the lecture table is illuminated by a bank of lights in the ceiling backed by reflectors directed toward the table. The ceiling lights of the room are divided into groups which may be controlled individually or by the relay switches that control the entire system. In this way one or more lights may be left on, thus permitting the class to see the lecturer perform an experiment and at the same time have the room darkened enough to render visible the spot from the galvanometer.

Both large and small lecture rooms are clear of supporting posts. The rear corners of the large room, which are inefficiently used in such a room, are given over to storerooms (see Fig. 2).

**Undergraduate laboratories**

Our undergraduate laboratory sections consist of 12 to 16 students and are assigned to separate rooms. A graduate assistant is in charge of each section. The laboratory rooms for mechanics, heat, light and electricity are in groups of three. Each room will have equipment for one experiment at a time. Students usually work individually but, for some experiments, in groups. In this way three experiments, in mechanics, for example, may be taught simultaneously. The students in the course rotate from room to room for the various experiments.

In the 12 rooms as many as 192 students may be accommodated at one time, although until recently the sections have been restricted to 12 students each, a total of 144 at one time. No set of rooms has been designated as acoustic laboratories. Experiments in sound are performed in the optics or heat laboratories.

Some of the apparatus is permanently attached to the walls of the rooms but the larger part of it is kept in storerooms until it is brought out for use. It is then placed on movable tables or, in several of the laboratories, on brick piers.

The windows of the optics laboratories are equipped with lightweight fiber doors placed within the window frame and opening inward. These can be used to darken the room.

Gas, air and electric outlets are placed at numerous points about the walls and also in the floor at table locations in all of the laboratories. The electric outlets are the standard gang of a.c., d.c., two low voltage battery and two high voltage battery outlets which are described later.

The intermediate laboratories are on the basement floor and adjacent to the elementary laboratories on the second and third floors. The larger of these "Modern Experimental Laboratories" have a number of large piers. Those on the upper floors are especially equipped for the various subjects; for example, the optics laboratories have adjacent dark rooms.

Our plan of instruction does not require many recitation rooms. One large one on the first floor and the seminar rooms suffice.

**Research facilities**

As indicated in the floor plans, nearly all the research rooms are located on the basement floor. The level of this floor is about 5 ft below the grade line, which permits some lighting by sunlight through light-wells outside the windows.

Most of the research rooms are single, but a few are large enough for several projects or a project requiring considerable space. The choice of room sizes seems to have been a happy one, as no research project has developed which could not be accommodated. A recent demand was met by the vertical shaft. Several experiments have been carried out in it, but it recently proved most satisfactory for a long column in which isotopes are separated by thermal diffusion.

All of the research rooms are equipped with brick piers with Alberene tops. These are table height and are of two sizes, 32×32 in. and 32×60 in. The piers are equipped with water supply, small drains, gas, air and one of the

standard electric outlet panels with a.c., and d.c. outlets, two outlets for low voltage circuits and two outlets for high voltage work. Figure 3 shows a side of a pier in use.

Placed along the walls of the laboratories and research rooms are channel irons into which bolts may be slipped at various points. There are usually two such horizontal channels placed about 3 ft apart and running the entire length of the wall. We have found them extremely convenient for attaching apparatus to the wall. Very rarely has it been necessary to dig holes in the tile walls. All workers in the laboratory have been very enthusiastic about this arrangement. Figure 4 shows a small shelf and the method of attachment. These channel irons are strong enough to support heavy apparatus or extensive shelves, as in Fig. 5. We have also set bolts into the concrete joists. These bolts extend a few inches below the plaster and are placed above piers and at other likely points about the research rooms and undergraduate laboratories. They have proved quite convenient for attaching apparatus. Thimbles have been placed in the tile walls between many of the rooms to permit temporary electric circuits or other connections to be made.

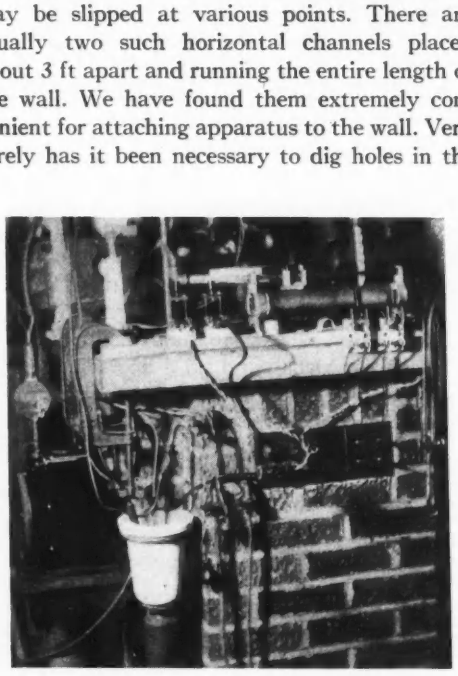


FIG. 3. Typical pier in research rooms. Electric outlets, drain, gas, air and water outlets are seen on the side just below the Alberene top.

**Electric equipment**

In a sub-basement room we now have 5 motor generator sets: 5 kw, 110 v; 15 kw, 110 v (these are driven by synchronous motors); 5 kw, 250 v; 15 v, 100 amp, for charging storage batteries about the building; and 3.5 kw, 750 v, for charging high voltage batteries. These are all connected with, and controlled from, a central power switchboard on the basement floor, from which the current is distributed to the various parts of the building.

The battery switchboard (Fig. 6) is unique. It is essentially a large bank of bus-bars extending entirely around the room. The battery terminals are at the base of the board. The terminals are two clips such as are used for cartridge fuses. Into these, vertical connectors may be placed. Several are shown in Fig. 6. These connectors extend upward to any pair of bus-bars that may be chosen. The upper ends of the connectors have clips, such as are used in knife switches, and fit over the bus-bar. The room circuits terminate at the top of the board in fuse clips like those at the bottom of the board. The upper connectors are similar to those used below except that they are inverted and are double. The upper connectors—"jumpers," we call them—are made of two brass tubes attached to insulating blocks at each end. The lower connectors are similar single tubes. Such an arrangement has great flexibility. Any cell or group of cells may be connected to any circuit. Only one set (two pairs) of connectors, or jumpers, of complementary lengths are available, so that if a set is on the storage rack we can be sure that the bus-bars to which they will reach are not in use.

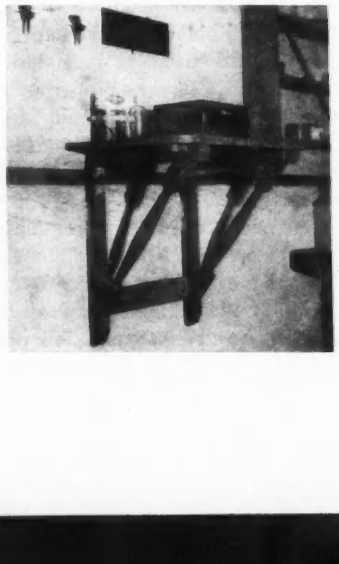


FIG. 4. Channel irons placed in the walls to support shelving, instruments, etc. An electric outlet, and gas and air lines are shown above the shelf.

As originally wired, each cell (2 v) in the low voltage bank and each group of cells (50 v) in the high voltage set had a separate pair of leads, and switches were placed between each pair of leads so that the circuit could be insulated from the rest of the board. Such an arrangement introduced a large amount of resistance. This has been reduced by connecting the low voltage cells into groups of three directly at the cells. The arrangement gives a very neat and flexible switchboard without a mass of wires as is necessary when conical plugs are used. Our switches (a form of snap switch) were none too good and the combination does give a high line-resistance. Some of the switches have been replaced by the knife type.

The standard electric outlets mentioned earlier are distributed generously throughout the building. These small panels contain four conventional duplex receptacles like those used in a residence. One is connected to the 110-v, a.c. lines, another to the power switchboard and can then be connected to 110-v d.c. or a.c. or 220-v d.c. or a.c. Two circuits run to the low voltage battery. Two

FIG. 5. This shows how shelving, a switchboard and similar items can be attached to the wall without cutting holes in the tile.

additional circuits from the high voltage battery board terminate in a special plate with pin-and-jack connections. In the research rooms one or more such panels are placed on the walls and one on the pier. In the smaller rooms these will be in parallel. In the larger rooms there are several groups with independent circuits. One of the outlets is shown in the upper part of Fig. 4. A distinct disadvantage of these outlets is that all

FIG. 6. The battery switchboard. Less than one-half of the low voltage board is shown. A similar board is installed for the high voltage batteries.

circuits to the rooms contain the variable and relatively high resistance of the receptacle. A better plan would be to have the circuits from the main switchboard terminate in each room at a small board with substantial binding posts and switches; from this board circuits could be run to outlets in various parts of the room such as we now have.

Heavy current circuits, 200 amp capacity, run to a number of the research and advanced laboratories. These terminate in heavy knife switches. Either d.c. or a.c., 110 or 220 v may be placed on these lines.

The batteries in the sub-basement consist of 110 cells (220 v) of high ampere-hour capacity, and 3000 cells (6000 v) of low ampere-hour capacity. As the high voltage cells become old and useless they will be replaced largely by electronic devices.

Nearly all the wiring is placed in closed conduits. This makes a very neat and safe installation, but it also makes it difficult and expensive to add or reroute circuits. Such changes could be made much easier if the wiring, at least much of it, were placed in channels that could be opened.

**Shops**

The shops are in the basement and include a wood shop and metalworking shop open to graduate students and staff. A mechanic is on duty in these rooms to aid those who bring in work and also to build and repair apparatus for the student laboratories. The general University instrument shops are also in our building and occupy three rooms. One of the men in these shops is assigned to the Physics Department. A third mechanic or research assistant and the University glassblower have rooms near the general shops. All of the physics shops are served by adjacent stock rooms for metals, wood and glass. The shop rooms are crowded, and we hope

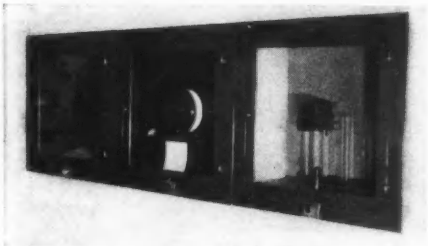


FIG. 7. Display cabinet in corridor which may be operated by students.

to assign additional rooms in the basement or sub-basement to the shops.

**Heating and ventilating**

The offices and smaller rooms are heated by radiators in the rooms. The lecture rooms have individual heating and ventilating systems. The air is heated and washed in the fan room (sub-basement) and circulated to the lecture rooms. The system for each room is separate from all other parts of the installation and may be controlled by switches at the lecture table. The heated air enters through registers near the ceiling and is withdrawn through "mushrooms" under the lecture seats. The noise level is low and the operation of the system disturbs no lecturer.

The ventilating system for other parts of the building is divided into sections, each with its own fan in the sub-basement. This permits a very flexible control. Special fans are furnished for hoods and for the battery room.

**General features**

The interior tile walls are all double with a clear space of about 2 ft between them in which are placed all the conduits, water pipes and ventilating shafts. Although the corridors, laboratories and offices are therefore entirely clear of these conduits, etc., the plan does result in considerable space between these walls which is not used. We are now building into such clear space storage cabinets which open with slab doors onto the corridors. A number of them have been completed in the corridor on the second floor and have been found very satisfactory.

The corridor of the first floor has a number of display cabinets (Fig. 7) containing various experiments that can be operated by the public. These cabinets are placed in the space between the two tile walls which we find ample for many interesting experiments. Our lecture rooms are used by outside groups as well as by physics classes, and many students, while waiting for classes, entertain themselves by operating these devices.

In a recently constructed building on our campus the space in the double walls has been recessed where that is possible and in this space a long rail with coat racks is placed. Such an arrangement is convenient in Minnesota, for otherwise a part of the seats in any lecture room in the winter must be given over to coats and wraps.

In the sub-basement two large rooms have been finished and are used as a "morgue." Old research equipment, lecture and laboratory apparatus which are no longer in use, and odd pieces

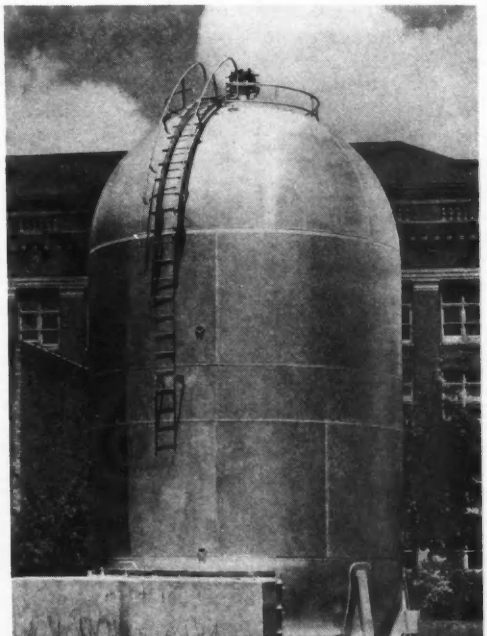


FIG. 8. The high voltage generator of the Van de Graaff type. The steel tank permits operation at air pressure of 100 lb in.$^{-2}$.

of stock are placed in these rooms. We have found them extremely useful and they have justified their expense many times in the saving of space in other parts of the building and in the saving of time and materials that would have been spent on new apparatus if the old had not been available.

The building has been modified slightly for the installation of a high voltage Van de Graaff generator at the rear of the building (Fig. 8). The control room and apparatus room below the steel tank are entered from the basement.

An astronomical dome has been added to the roof (Fig. 1) since the building was constructed. A portion of the third floor and the roof house (not shown in Fig. 2) is given over to the department of astronomy.

Our building has been found most satisfactory. Desired changes have been made, for the most part easily. We may characterize the building as "versatile"; it adapts itself to the continually changing trends of research. For its plan, the Department of Physics is most grateful to Dr. Henry A. Erikson, now Professor Emeritus. It was due to his foresight, in planning for many years in advance of the appropriation of funds, that we have such a building. When the funds were available, the plans were also very quickly put into final form. It is interesting to note that this is the second building for physics on this campus which Doctor Erikson helped plan and on which he supervised construction; the previous building, completed in 1904, is now used for other purposes.

## Considerations on Ballast Resistors

WILLIAM R. HASELTINE
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ONE of the simplest means of stabilizing the current in a load against fluctuations in the line voltage is the use of a hot wire ballast resistor. The theory of the operation of this device is sufficiently elementary that its derivation or application might be used as a problem in an intermediate course on electricity. In fact, the problem is presented in an oversimplified form in Loeb's *Fundamentals of Electricity and Magnetism*.[1] Since an analysis of the operation of the ballast resistor does not seem to be readily available, the following discussion of it is presented.

The primary characteristic of the device is simply a resistance which increases with the power dissipated in it, so that the current through the wire increases less rapidly than the potential drop across it. As usually built, including commercial designs, the ballast consists of an iron filament in an atmosphere of hydrogen. It is run at a dull red heat.

In the example in the text referred to, the problem is stated essentially as follows. A wire has a resistance given by

$$R=R_0(1+\alpha t) \tag{1a}$$

and dissipates energy at a rate

$$P=P_1 t, \tag{1b}$$

where $t$ is the difference between the temperature of the wire and room temperature. If $I$ is the current and $V$ is the potential drop, it is desired to find the current for which the quantity $\mathrm{d}I/\mathrm{d}V$ is zero. This would be the point at which stabilization is complete. From the equations

$$V=IR, \tag{2a}$$

$$P=VI=I^2R \tag{2b}$$

and Eqs. (1), one can eliminate $t$, obtaining $I$ as a function of $V$ (or vice versa). Thus, explicitly,

$$V=R_0I/(1-\alpha R_0I^2/P_1). \tag{3}$$

For $I=\sqrt{(P_1/\alpha R_0)}$, $\mathrm{d}I/\mathrm{d}V$ is, it is true, zero; but for the same value, $V$, according to Eq. (3), becomes infinite.

This unsatisfactory indeterminacy is a result of the oversimplification inherent in the forms assumed for $R$ and $P$ as functions of $t$. Consider

[1] L. B. Loeb, *Fundamentals of electricity and magnetism* (Wiley, ed. 2, 1938), p. 479.

the more general forms,

$$R=R_0(1+\alpha_1 t+\alpha_2 t^2+\cdots), \tag{4a}$$

$$P=(P_1 t+P_2 t^2+\cdots). \tag{4b}$$

If the higher order terms are not neglected, the indeterminacy does not arise. For example, if one writes

$$R=R_0(1+\alpha_1 t+\alpha_2 t^2), \tag{5a}$$

$$P=P_1 t \tag{5b}$$

and rejects all higher order terms as negligible, one obtains the following.

From Eqs. (2),

$$I^2=P/R, \tag{6a}$$

$$V^2=PR \tag{6b}$$

or

$$I^2=P_1 t/R_0(1+\alpha_1 t+\alpha_2 t^2). \tag{7}$$

Now

$$\frac{dI}{dV}=\frac{dI}{dt}\Big/\frac{dV}{dt}. \tag{8}$$

Taking derivatives with respect to $t$,

$$2I\frac{dI}{dt}=\frac{P_1 R_0(1-\alpha_2 t^2)}{R_0^2(1+\alpha_1 t+\alpha_2 t^2)^2}. \tag{9}$$

At such a temperature, then, that

$$t=\sqrt{(1/\alpha_2)}, \tag{10}$$

$dI/dt$ will be zero, and hence from Eq. (8), $dI/dV$ will also be zero. Under these conditions $R=R_0(2+\alpha_1/\sqrt{\alpha_2})$, and $P=P_1/\sqrt{\alpha_2}$.
The current in this state is given by

$$I=P_1/R_0(2\sqrt{\alpha_2}+\alpha_1). \tag{11}$$

This is an ideal case, of course, and if higher order terms are required, the solution is less simple. In particular, it is not always possible to find a current such that $dI/dV=0$. (This is impossible even in the foregoing case if $\alpha_2$ is negative.) Furthermore, the resistance and power characteristics of a given filament would be difficult to obtain. It is, however, easy to obtain the resistance of a given device as a function of the power dissipated in it; this requires only a simple series of current and potential measurements. It is, therefore, convenient to use the power, rather than the temperature, as the independent variable. It is also convenient to change the point of view to that described in the next paragraph.

Consider a load resistance $R_L$, taken as constant. Let the current be $I$, the potential drop be $V_L$ and the power developed be $P_L$. Then $V^2=P_L R_L$, $2V_L dV_L=R_L dP_L$ and

$$dP_L/P_L=2dV_L/V_L. \tag{12}$$

This states that, for small changes, the proportional change in the power is twice the proportional change in the voltage. If a ballast be placed in series with the load, and the voltage across both be $V$, one will find that

$$dP_L/P_L=\gamma dV/V. \tag{13}$$

If the ballast is so chosen that $\gamma$ is very small, then the proportional change in the power developed in the load is much smaller than the proportional change in line voltage, and stabilization is attained. The amount by which $\gamma$ is smaller than two is a measure of this stabilization.

Let $R_b$ be the resistance of the ballast, $P_b$ the power dissipated in it and $V_b$ the potential drop across it. Let $R_b$ be regarded as a function of $P_b$. Then

$$V^2=(R_L+R_b)(P_L+P_b), \tag{14}$$

$$I^2=P_L/R_L=P_b/R_b$$
$$=(P_L+P_b)/(R_L+R_b), \tag{15}$$

$$dP_L/R_L=dI^2=(dP_b/R_b)-(P_b dR_b/R_b^2) \tag{16}$$

or

$$dP_L/R_L=\frac{1}{R_b}\left(1-\frac{P_b}{R_b}\frac{dR_b}{dP_b}\right)dP_b$$
$$=(1-X_b)\frac{dP_b}{R_b}. \tag{17}$$

This defines

$$X_b=(P_b/R_b)(dR_b/dP_b). \tag{18}$$

Differentiating Eq. (14),

$$2V dV=(R_L+R_b)(dP_L+dP_b)$$
$$+(P_L+P_b)(dR_b). \tag{19}$$

Using Eqs. (15) and (16), one obtains by simple algebra,

$$\frac{2dV}{V}=\frac{1+X_b(R_b-R_L)/(R_b+R_L)}{(1-X_b)}\frac{dP_L}{P_L} \tag{20}$$

or

$$\frac{dP_L}{P_L}=\frac{2(1-X_b)}{1+X_b(R_b-R_L)/(R_b+R_L)}\frac{dV}{V}=\gamma\frac{dV}{V}. \tag{21}$$

Equation (21) says that when $X_b$, or $(P_b/R_b)(dR_b/dP_b)$, is nearly equal to one, $\gamma$ is very small and the stabilization good.

The resistance $R_b$ can be obtained as a function of $P_b$ for a given ballast, and $dR_b/dP_b$ can then be found, by graphical differentiation if necessary. Thus the range in which this device will give stabilization is ascertained, and its usefulness for the purpose in hand determined.

If the load resistance also varies with the power developed, this must be taken into account. One can define, analogously to Eq. (18),

$$X_L=(P_L/R_L)(dR_L/dP_L) \tag{22}$$

and then obtain

$$\frac{dP_L}{P_L}=\frac{\gamma dV}{V}$$
$$=\frac{2(1-X_b)}{1-X_bX_L+(X_b-X_L)(R_b-R_L)/(R_b+R_L)}\frac{dV}{V}. \tag{23}$$

The appearance of $X_L$ expresses the effect of a variable load resistance in decreasing the stabilization.

In any case, it is of advantage to have $R_b$ considerably larger than $R_L$, for this increases the term $(X_b-X_L)(R_b-R_L)/(R_b+R_L)$ occurring in the denominator.

The curves of Fig. 1 illustrate these equations. The data were obtained in the electrical engineering laboratory through the kindness of Professor L. C. Marshall. The ballast used was an RCA 886 tube, and the load was the heater of a type-80 tube. The nominal rating of the 80-heater is 2.0 amp, 5.0 v. The 886 tube is supposed to provide stabilization at 2.05 amp, with a drop of 40 to 60 v across it. Curve *1* shows $P$, the load power, plotted as a function of the line voltage $V$. Curve *2* shows $\gamma$ as calculated directly from curve *1*. The derivative $dP_L/dV$ was taken graphically. To obtain curve *3*, $R_b$ and $R_L$ were plotted against $P_b$ and $P_L$, respectively. Derivatives were taken graphically, and $\gamma$ calculated according to Eq. (23). The agreement between curves *2* and *3* is probably as good as can be expected, considering the uncertainties of evaluating the derivatives. They show at least that the performance can be predicted to better than a merely qualitative degree. It should be noted that $X_L$ was very small in this case, or, in other words, that the load resistance was nearly constant but not completely so. The data were obtained using 60-cycle a.c.

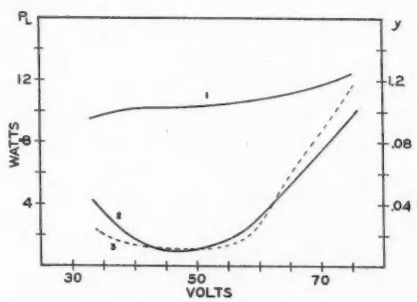


FIG. 1.

The usefulness of iron as a stabilizing element may be due to the fact that it has a transition point at about 700°C, near the operating temperature. One would expect a marked change in the resistance characteristic at such a point. These changes should show up most markedly in the values of $dR_b/dP_b$ as a function of $P_b$. As actually found, this derivative rises rapidly through most of the region where stabilization is effective, and shows a rather sharp break to lower values near the upper end of the range. No attempt has been made to correlate this with temperature, but it is clear that the electric properties of the wire are changing rapidly throughout the stabilized regime.

The minimum value of $\gamma$ is here about 1/11. This is probably about as large a degree of stabilization as can be obtained by this means. (This is a stabilization factor of 22.) It should be pointed out that there is one serious fault in this method of control. When the line voltage changes, there is a lag of as much as 1 min before the temperature of the ballast changes sufficiently to give maximum compensation. The method is still of value, however, in reducing fluctuations and as a protective measure.

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*WE stand, today, with our mechanical foot in an airplane and our social foot on an ox-cart.*—HARRY ELMER BARNES.

# The Prenatal Existence of the Reflecting Telescope

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THE reflecting telescope in its present form dates from the seventeenth century. The credit for first describing it goes to Niccolo Zucchi as of 1652 and for first constructing such a telescope to Isaac Newton as of 1668. There seems to be no tendency to question the priority of these men. But, at least in science and technology, major developments seldom spring full-fledged into existence. Hence it is no detraction to identify the early ideas and the rudimentary devices which precede great discoveries.

There is some indication that concave mirrors were in use as telescope objectives as long as three hundred years before the time of Zucchi and Newton. There is no evidence that magnifying glasses were used to view the images formed by these objectives, though the properties of converging lenses used as magnifiers are described in detail, albeit not entirely correctly, in some of the writings of that period,[1] and a few such lenses are known to have been made and used.[2] If clear evidence should ever be adduced that eyepieces were utilized in conjunction with the concave mirrors known to have been constructed and used at that time, it would be necessary to revise the prevailing idea of the date of birth of the reflecting telescope. It is very probable, however, that the eyepiece, as an adjunct to a telescope objective, was not devised until much later, the concave-mirror objectives being originally made to do duty without eyepieces. Telescopes can be so used, fairly effectively, as many physicists can testify out of incidental experiences in the laboratory.

At one time the thirteenth-century friar, Roger Bacon, was erroneously credited, in a famous mistranslation, with having been the first to describe a *refracting* telescope. The error was made by the translator, of course, and is not to be held against Bacon. In any case, it was corrected as long ago as 1890[3] and has no direct bearing upon the present paper except for the reaction which such episodes inevitably produce, creating a presumption against the validity of any further claims for the party involved. Such a presumption might come into play against the indications, presently to be presented, that Roger Bacon, among others, made and used concave mirrors in the way just suggested. Let it be clear, at least, that it is fully recognized that the evidence against Bacon's association with the refracting telescope is conclusive on several counts.

The documentary evidence offered herewith on the work of Bacon and others with concave mirrors prior to the actual birth of the reflecting telescope is not new. Moreover, no single one of the references can be considered more than mildly indicative. They were encountered one at a time in the course of some recent studies on the history of optics. At first paying little attention to them, the writer presently found himself somewhat impressed by the convergency of some six lines of evidence from independent sources. Though their content is meager, as is almost all historic material of that period, yet the prominence of their common factor suggests that here may be an episode in the many-sided life of Roger Bacon which has not received the attention that it merits. The lines are as follows.

First, in his *Opus Tertium* (pages 46, 112, 116) Bacon tells of his efforts to construct large concave mirrors. The first mirror was completed in 1267 or soon thereafter. He and his assistant, the famous Petrus Peregrinus, devoted an immense amount of time and effort to this undertaking. After working no less than three years almost exclusively on this labor and expending the sum of 60 Parisian pounds (the modern equivalent of about $2000), Bacon at last succeeded in making a mirror with which he was satisfied. Unfortunately he does not tell what his criterions of excellence were, nor what he did with his mirror when it was finished. Apparently

[1] Roger Bacon, *Opus majus*, tr. by R. B. Burke (Oxford Univ. Press, 1928), vol. II, pp. 569, 574.

[2] Roger Bacon, *Opus tertium*, ed. by J. S. Brewer (Longmans Green, 1859), p. 111.

[3] Wiedemann, Wiedemann's Ann. **39**, 130 (1890).

he made others, for he says:

> The first mirror cost sixty Parisian pounds, and afterward I managed to make a better one for ten Parisian pounds, and after that, as a result of my diligent experimentation with them, I perceived that still better ones could be made for two marks or even for less.

BACON does not refer again explicitly to these mirrors, but at various points in three of his other publications[4] which, taken together, may be termed the second of our six lines of evidence, he tells of his observation that the focal point of a concave mirror is half the radius away for rays in the region of the axis, but that it is much nearer the mirror for rays reflected from the remoter zones. If this originates in anything more than geometry, he could scarcely have observed such a phenomenon without a rather high quality mirror at his disposal.

The next time we hear of BACON's mirrors is more than a century later in the writings of one *Peter of Trau* in Dalmatia in the year 1385, which will be considered our third line of evidence. He states that

> Friar Roger, called Bachon, an Englishman . . . made two mirrors in the University of Oxford; by one of them you could light a candle at any hour, day or night; in the other you could see what people were doing in any part of the world.[5]

In these statements, notwithstanding their obvious exaggerations, are evident the burning-glass and at least a rudimentary predecessor of the reflecting telescope. PETER goes on to say that the students of Oxford spent so much time experimenting with these mirrors, to the detriment of their studies, that both mirrors were broken by order of the university authorities. Evidently the problem of extracurricular activities is not of recent origin.

Commentators are inclined to belittle this account of 1385 because of its close juxtaposition with a story, by the same author, in which BACON is said to have constructed a bridge of solidified air and then caused it to disappear again after he had crossed over it. The bridge episode and the account of the two mirrors appear in the same paragraph, though they are not textually connected. Despite the implication that both are made up out of the same cloth, it does not follow that the cloth was entirely woven of imagination. One wonders whether the good PETER was not relating the experience of someone who had been the victim of an optical illusion by one of the mirrors. How might writers of that time have described, for example, the classical modern mirror demonstration, in which the real image of a bouquet of flowers is placed in a corporeal vase and made to appear and disappear at will by controlling the illumination?

The fourth reference to BACON's mirrors was in 1551, in a book entitled *Pathway to Knowledge* written by ROBERT RECORDE. RECORDE was court physician to Edward VI. He was a mathematician of some contemporary note, a lecturer on geometry at Oxford. In his preface, commenting on the fact that the vulgar ignorant are inclined to attribute all unfamiliar occurrences to magic, he used BACON and his mirrors as an example:

> And hereof came it that fryer Bacon was accompted so greate a negromancier, which neuer vsed that arte. . . . Greate talke there is of a glasse that he made in Oxforde in which men myght see thynges that were doon in other places, and that was iudged to be doon by power of euyll spirites. But I knowe the reason of it to bee good and naturall and to bee wrought by geometrie . . . and to stand as well with reason as to see your face in common glasse.

For the fifth line of evidence we have to go only 20 years beyond the time of ROBERT RECORDE, which leaves still 80 years before the first explicit description of the reflecting telescope by ZUCCHI. THOMAS DIGGES, a military commentator, tells in a book entitled *Pantometria* (Preface, edition 1, 1571) of the advantages of certain optical devices worked out by his father, LEONARD DIGGES.

> My father, by his continual paynful practises, assisted with demonstrations Mathematicall, was able, and in sundry times hath, by proportionall Glasses duly situate in conuenient angles, not onely discouered thinges farre off, read letters, numbered peeces of money with the very coyne and superscription thereof, cast by some of his friends of purpose vppon Downes in open fieldes, but also, seuen myles of[f] declared what hath been doon at that instant in priuate places. He hath also sundrie times by the Sunne beames fired Powder and discharde Ordinaunce halfe a myle and more distante, which thynges I am the boulder to reporte for that there are yet liuing diuerse Oculati testes (of these his dooings).

[4] See E. Hoppe, *Geschichte der Physik*, p. 249.
[5] A. G. Little, Proc. Brit. Acad. **14**, 265 (1928).

And again, in 1579, in a second book entitled *Stratiotikos* (edition 1, page 89), DIGGES recounts similar exploits of his father and continues:

> As sithence Archimedes (Bakon of Oxforde onely excepted) I haue not read of any in Action euer able by meanes naturall to performe ye like. Which partely grew by the aide he had by one old written booke of the same Bakon's Experiments, that by straunge aduenture, or rather Destinie, came into his hands.

Though DIGGES does not explicitly mention the concave mirror, his second comment follows a section setting forth periscopic uses of plane mirrors in surveying. Reflection is certainly implied as the means by which these effects were secured. If these things were actually done as alleged, then the elder DIGGES has the distinction of having used some kind of rudimentary telescope, presumably of the reflecting type, fully a century before the complete reflecting telescope came into being, whether any credit be considered due to BACON or not.

The sixth and last line of evidence the writer approaches with trepidation. It has to do with a document which has suffered severely from apparently excessive claims that have been made as to its content. Fortunately there seems to be no difference of opinion about the likelihood of its being an authentic thirteenth-century document. It therefore takes us back again to the time of BACON. There seems to be a certain amount of doubt whether it is actually from his pen, though even the most refractory critics do not raise much of an issue against that contention. The document is the so-called Voynich manuscript, the principal study of which was made by the late PROFESSER NEWBOLD of the University of Pennsylvania. NEWBOLD'S work on this manuscript, quite incomplete at the time of his death, was, nevertheless, collected and posthumously published in 1928 under the title, *The Cipher of Roger Bacon*. The storm of criticism which this publication aroused[6] indicates that a better service might have been done the memory of the author by allowing his work to remain unpublished. The chief objection seems to be that NEWBOLD'S method of decoding the cipher was so loose that it allowed almost any interpretation, and that the author did, in fact, allow his enthusiasm and fertile imagination to supply much in the translation for which there is little warrant in the original.[7]

Fortunately, very little dependence upon NEWBOLD'S rendition is necessary. At one place in the manuscript there is an illustration which, to anyone acquainted with physical science, clearly suggests a spiral nebula. NEWBOLD translates the accompanying text in part as follows: "Vidi stellas in speculo concavo, in cochleae forma agglomeratas." NEWBOLD is certainly wrong in his accompanying speculation that the writer of the manuscript was observing and describing the *Andromeda* nebula. A conclusion that it was some unidentified spiral nebula would have been less untenable, in view of the support furnished the text by the illustration, but even that is only an incidental item from our present standpoint, albeit an item of considerable interest. The important thing is that at some very early time, probably in the thirteenth century, someone, whoever he may have been, used a concave mirror as a visual aid in stellar observation. Such an episode, if clearly established, would help materially in identifying a stage in the prenatal development of the reflecting telescope.

This concludes the six references which were mentioned at the outset. Perhaps it should be repeated that no single one of them can be considered as more than mildly indicative, and perhaps all of them taken together are not more than that. These six lines do not exhaust the known evidence on the point at issue, but seem to be the least equivocal. Moreover, even if they were as circumstantial and conclusive as could be desired, they would, to judge from their trend, merely make clearer some stages leading up to the development of a very useful tool, the credit for the actual first construction of which is in no way called into question.

[6] Manly, "Roger Bacon and the Voynich Ms.," Speculum **6**, 345–9 (1931); G. Sarton, Isis **11**, 141 (1928); A. G. Little, Proc. Brit. Acad. **14**, 265–6 (1928).

[7] Without maintaining that these criticisms are unjustified, one may remark that the depth of invective in which one of the critics has permitted himself to indulge can scarcely act to attract scholars to a field in which that same critic elsewhere deplores the lack of activity. The criticism involves such terms as "idiotic thing," "cranks," "Baconian fools" [G. Sarton, Isis **11**, 144 (1938)]. Compare with Sarton's *Introduction to the history of science*, vol. II, part 2, p. 964: "The lack of a complete edition of Bacon's work is a real shame."

# Reproductions of Prints, Drawings and Paintings of Interest in the History of Physics

## 14A. Paintings of Lectures at the Royal Institution: Michael Faraday Lecturing on "Metals," December 27, 1855

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A SPECIAL course of six Christmas lectures adapted to a juvenile audience has been given at the Royal Institution of Great Britain (see reproduction 11 in this series) every year since 1826. These lectures were originated by Michael Faraday (1791–1867) who himself gave no less than nineteen of them. His subject for 1855–56 was "The distinctive properties of the common metals," and the first lecture of the series was evidently an occasion of importance, for the scene was sketched and afterwards painted by Alexander Blaikley (1816–1903). Plate 1 shows the original sketch as it was reproduced in the *Illustrated London News* of February 16, 1856, while Plate 2 is from the painting that now hangs in the rooms of the Royal Institution. Prince Albert, the Royal Consort, is shown in the center of the first row of the audience with the Prince of Wales (afterwards King Edward VII) at his right and Prince Alfred (the Duke of Edinburgh) on his left. The tallest figure in the first row at the right of the lecture table is Henry Bence Jones, Faraday's biographer and the secretary of the Institution from 1860 to 1873. In the fourth row are shown John Tyndall (immediately behind Prince Alfred) and Mrs. Faraday (close to the middle of the picture and directly in line with the center of the lecture table). Many of the other figures have also been identified.

Faraday's skill as a lecturer is well known, but it may be of interest to quote the tribute of Lady Pollock,[1] a frequent attendant at his lectures.

> He [Faraday] should be remembered in his characteristic phases; first as he stood at the lecture table, with his voltaic batteries, his electro-magnetic helix, his large electrical machine, his glass retorts, and all his experimental apparatus about him,—the whole of it being in such perfect order that he could without fail lay his hand upon the right thing at the right moment, and that, if his assistant by any chance made a blunder, he could, without a sign of discomposure, set it right. His instruments were never in his way, and his manipulation never interfered wth his discourse. He was completely master of the situation; he had his audience at his command, as he had himself and all his belongings; he had nothing to fret him, and he could give his eloquence full sway. It was an irresistible eloquence, which compelled attention and insisted upon sympathy. It waked the young from their visions and the old from their dreams. There was a gleaming in his eyes which no painter could copy and which no poet could describe. Their radiance seemed to send a strange light into the very heart of his congregation; and when he spoke, it was felt that the stir of his voice and the fervour of his words could belong only to the owner of those kindling eyes. His thought was rapid, and made itself a way in new phrases, if it found none ready made,—as the mountaineer cuts steps in the most hazardous ascent with his own axe. His enthusiasm sometimes carried him to the point of ecstasy when he expatiated on the beauty of nature, and when he lifted the veil from her deep mysteries. His body then took motion from his mind; his hair streamed out from his head, his hands were full of nervous action, his light, lithe body seemed to quiver with its eager life. His audience took fire with him, and every face was flushed. Whatever might be the after-thought or the after-pursuit, each hearer for the time shared his zeal and his delight; and with some listeners the impression made was so deep as to lead him into the laborious paths of philosophy, in spite of all the obstacles which the daily life of society opposes to such undertakings. . . . There are instances where a strong effect is produced by a speaker who is conscious of it and who strives for it; but with Faraday the effect was due to his unconsciousness, to his forgetfulness of himself, and to the concentration of all his intellect and all his emotion upon the thing he was teaching.
>
> A pleasant vein of humour accompanied his ardent imagination, and occasionally, not too often, relieved the tension of thought imposed upon his pupils. He would play with his subject now and then, but very delicately; his sport was only just enough to enliven the effort of attention. He never suffered an experiment to allure him away from his theme. Every touch of his hand was a true illustration of his argument. Foreigners, children, and fine ladies felt as if they understood what he told them,—partly because of the simplicity and sincerity of his manner, and partly because he excited their enthusiasm so much that they did not question their

[1] *The Saint Pauls Magazine*, June, 1870, p. 293–295.

PLATE 1. FARADAY LECTURING AT THE ROYAL INSTITUTION

(From the original sketch of Alexander Blaikley as reproduced in the *Illustrated London News*.)

understanding. But his meaning was sometimes beyond the conception of those whom he addressed. When, however, he lectured to children, he was careful to be perfectly distinct, and never allowed his ideas to outrun their intelligence. He took great delight in talking to them, and easily won their confidence. The vivacity of his manner and his countenance, and his pleasant laugh, the frankness of his whole bearing attracted them to him. They felt as if he belonged to them; and indeed he sometimes, in his joyous enthusiasm, appeared like an inspired child.

Another contemporary account of FARADAY's lectures which appeared in *Punch* of March 14, 1857, is well worth quoting. It is one of "Mary Ann's Notions," a series of letters on current events supposedly written to *Mr. Punch* by "a fashionable young lady of Victorian tastes." All but the two introductory paragraphs are quoted in order to give the whole flavor of the letter:

> Do you know DR. FARADAY? I suppose so, as you know all the clever people in the world. Isn't he a dear? We went, that is LIZZY HAMERTON and her brother CHARLES, and AUGUSTUS and me, to the Royal Institution the other night, and DR. FARADAY gave a lecture. PRINCE ALBERT was there with his star on, looking so grave and elegant; and by the way, I do wish that you would not have ridiculous pictures made of him, for he is excessively good-looking still, and I dare say much handsomer than any of you folks that caricature him. He listened with the utmost steadiness, and I do not believe he moved half a quarter of an inch all the time. They set him in a great chair, you know, exactly in front of the lecturer. We had pretty good seats, considering that AUGUSTUS kept us waiting a quarter of an hour while he smoked his cigar (CHARLES HAMERTON don't smoke), but it is extremely absurd to see rows of old gentlemen, mostly with bald heads, in the front of the audience, and of course in the best places, while ladies are poked up in back rows. When AUGUSTUS came from school, he used to say something in Latin—ingenious diddy something—meaning that studying the arts and sciences hindered men from being Bears! I am sure it does not in Albemarle Street, or a couple of the old creatures would have given up their places to me and LIZZY.
>
> But the lecture was lovely. It was quite a treat to look at dear DR. FARADAY's earnest face and silvery hair, not that he is an old man, far from it, and he is far more

light and active than many a smoky stupid all-round collar-man that I know, and I believe that it is the cigar-smoke that makes you all so sluggish, and the doctors are quite right. CHARLES HAMERTON says that tobacco drives almost everybody mad, besides bringing on asthma, and blindness, and paralysis, and corns. I hope you don't smoke, my dear *Mr. Punch*, it would make me very miserable if I thought you did. But I was going to tell you about the lecture. Do you know what Gravitation is? Of course you will say you do. Well, it is all wrong, and so poor children are not to be bothered by Governesses with that rubbish any more. It is all—let me have the words right—it is all Conservation of Forces. This seemed quite clear to me at the time, especially with the beautiful experiments which he does so carefully and yet so easily. I am not certain that I can explain it quite so well now, but if you hit a piece of lead very hard, it sets fire to phosphorus; and if you stick up two pieces of iron, and sprinkle nails over them, they make a perfect rustic bridge. That is, you know, you must put them near an electrifying machine, and when you take away the wires, down comes the bridge, or the Tour de *Nesle*, as CHARLES HAMERTON cleverly said. Then if you take a long platinum wire, and electrify it, it becomes red hot in a single second, and you can make amethyst sparks fly out of it. This proves the Conservation of Forces, and it only shows what idiots men are to go on repeating gravitation, gravitation, like cuckoos, just because SIR ISAAC NEWTON saw an apple fall out of a tree (and I dare say he eat it, like a pig, as all men are) and now comes a really clever philosopher, and explains it all away. I could not tell you of half the experiments dear DR. FARADAY did, but there was one, when he rubbed a bit of sealing-wax in some flannel, and made some gold leaves dance in a jar, which proved quite clearly to me that there must be some force to do it, somewhere, because they never danced of themselves. It was a most beautiful lecture, and if anything could excel it, it was the kindness of DR. FARADAY afterwards, when ladies came and asked him questions, and he did not look supercilious, or what is worse, look condescending, but he entered with evident pleasure into explanations, and did several little experiments for us, electrifying some things like large metal buttons, and turning wet paper brown with them; and if we did not understand it, it was our own faults, not his, or rather it was the fault of the system of education you men give us, which makes us either quote like parrots, or stare like owls, when philosophy comes up.

Another thing struck me, and I must say it. Here was DR. FARADAY, a really great man, diving into the wonderful secrets of nature, and explaning them in the ablest manner. Where were all the great men and the statesmen, and the M.P.'s, and all those who pretend to lead

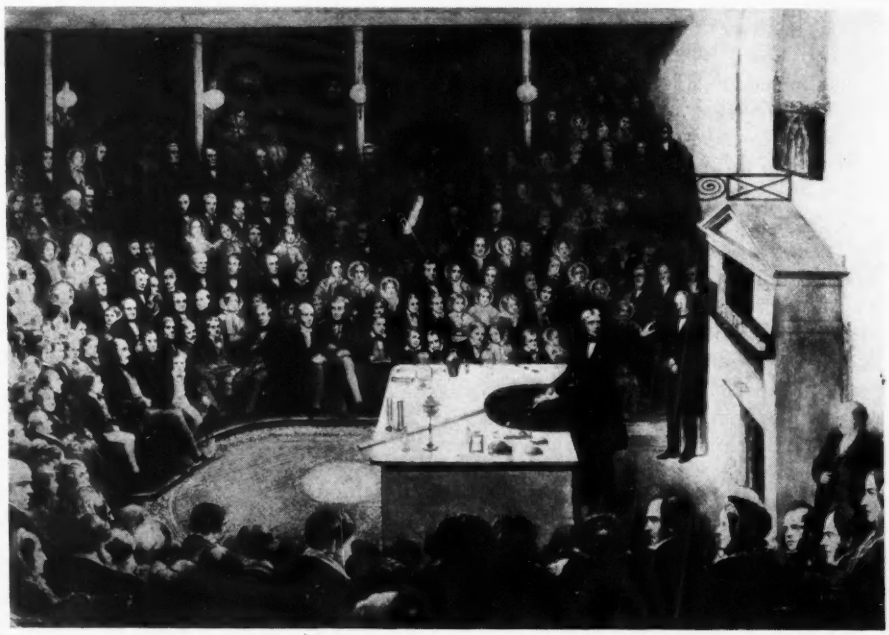

PLATE 2. FARADAY LECTURING AT THE ROYAL INSTITUTION

(From the painting by Alexander Blaikley which hangs in the rooms of the Royal Institution. By permission of the Managers.)

the world? Listening to him, as he unfolded these mighty things? Not they. That very night it seems, there was a fierce squabble going on in Parliament, nominally about the savages in China, but really to settle whether one set of H.'s (you know) or the other should have situations of Government, and take our money. And such is the nature of men that for one person in London who was thinking that night about DR. FARADAY and his splendid discoveries, a hundred were arguing and betting whether Ministers would be beaten or not. As CHARLEY HAMERTON (he told me this) said, very wittily, "I wonder whether LORD PALMERSTON will be as successful in his Conservation of his Forces." Dear LORD PALMERSTON, I consider the way he is persecuted as perfectly WICKED, and you may print that I say so.

"Yours, affectionately,"

"MARY ANN"

"*Monday*"

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# The Teaching of Physics in the Schools of England

## A BRIEF SURVEY OF ITS DEVELOPMENT AND ITS PRESENT POSITION

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### Brief history

PRIOR to the middle of the nineteenth century very little science was taught in the English schools—the subject being regarded as "foreign" to the superbly strong classical bias of the then existing curriculum. There is record, however, of some physics being taught in the larger public schools. For instance at University College School, London, about 1832, there was instituted a course that included some elementary experimental work in physics, although no definite laboratory seems to have been provided for it. About 1850 a room was set aside as a laboratory at the City of London School, and elementary physics was also being taught at King's Somborne School by Dean Dawes. In 1856, however, the Bristol Trade School was opened with a definite laboratory and workshop in which the boys made most of their own apparatus. About this time too, other schools began to realize that science was becoming important; and although they would not include it in their curriculum, they arranged for popular lectures on the subject by visiting lecturers.

Rugby was one of the first large public schools to build a special department of physics with adequate laboratories. This was in 1860. Prior to this, Canon Wilson—who later became Head of Clifton College, and who was in charge at Rugby—taught "in the cloak room on the ground floor of the Town Hall, which was furnished with tables and chairs and a cupboard containing apparatus for illustrating elementary objects, hydrostatics, mechanics and heat, and some good electrical apparatus." But in most schools the state of affairs was worse than this, as was discovered by the Public School Commissioners, set up in 1862 to investigate the position of science in this type of school. F. W. Westaway,[1] for instance, when he began teaching found that in his school there was "no laboratory and no fitted demonstration table. The only balance available was a homemade one costing about 3/6 with scraps of metal for the smaller weights. It weighed fairly accurately to one decimal place. The apparatus in stock was worth about two pounds."

But very gradually, almost imperceptibly, physics was beginning to push its way into the school curriculum. In 1868 the Taunton Commission, in its report, stated that, though there was great lack of laboratory accommodation in the endowed schools, a great deal of progress had been made by the public schools and the following schools were about to erect new laboratories: Harrow, Eton, Charterhouse, Christ's Hospital, Rossall, Wellington, Cheltenham, Taunton and Dulwich. At the same time there were several teachers who were doing real pioneer work. At Clifton College, A. M. Worthington arranged a definite school course in physics which catered mainly to the specialist going on to university work. But he did provide, also, for a large amount of laboratory work, even in the lower school. Though most of the apparatus was crude and

[1] F. W. Westaway, *Science teaching* (Blackie, 1929).

very simple, a high degree of accuracy was obtained and the course was formal and logical. Doctor Pears was working on similar lines at Repton School.

Physics at Harrow was in a very unhappy state until it was taken over by Sir Cyril Ashford. He relates that when he took over charge of the subject:

> . . . there was remarkably little Physics to take charge of. My predecessor told me that he had discouraged the teaching of science in the Upper School; although there was quite a good laboratory as well as a lecture room. Almost the whole of my work, which amounted to eleven periods a week, consisted in lecturing to a few divisions of the lower school. . . . My predecessor had been in the habit of giving courses of experimental lectures, on the lines of the Royal Institution Lectures. He had been probably influenced by the fact that Tyndall had been the Royal Society's representative on the Governing body, and had taken a keen interest in what little science teaching had been done at the school.

Tyndall, however, was not the only great scientist who took an interest in school science at that time. H. E. Armstrong, Professor of Chemistry, City and Guild's Institute, was having a very strong influence at Christ's Hospital School. He is well known as the originator of the "heuristic" method in science teaching, believing, as W. H. Fyfe[2] tells us, that "Natural Science should be taught without books. The basic facts of chemistry, physics and biology should be discovered by the pupils' own experiment and observation, and they should learn to express the results of their investigation in lucid English."

Such men, by their writings, fostered a great deal of interest in school science and provided a fillip to its development. However, headmasters —except such great men as Sanderson of Oundle —still looked upon physics with disfavor. For instance, a circular sent out to the Headmasters' Conference in 1877 asking how many of them were in favor of practical physics, produced only two answers in the affirmative out of eighteen who bothered to reply!

No great advance was made in the teaching of the subject until the year 1902. That year saw the introduction of the Education Act setting up new grammar and secondary schools, having as part of their curriculums the teaching of physics. The situation gradually improved; more and more teachers took up the subject and the university courses became filled to capacity. It was not until the year 1916, however, that the interest of the average man and woman was aroused in the position of science in the schools. The war had been on a year and its grim realities brought home to the citizens of the country the importance of science in their everyday lives. They began to realize that the subject had been neglected, and this theme was quickly taken up by the press. As a result, the position came before the eyes of Parliament; toward the end of 1916 the Prime Minister formed a committee under the chairmanship of Sir J. J. Thomson to investigate the matter, and its report was published in 1918. About this time the Board of Education became interested in higher courses, that is, courses after School Certificate stage, and offered grants for those students taking specialist courses in the Sixth Forms. This enabled more schools to arrange a higher course in science and also helped them to equip their laboratories with the expensive apparatus necessary for this type of work.

Mainly because of such steps as those taken in 1916, science, during the following years, enhanced its position considerably and, perhaps for the first time, became a vital, integral part of the intellectual activity of the nation.[3] The extent to which the subject had grown may be judged from the remarks of General Smuts in the Sidgwick Memorial Lecture of 1930:

> The rise of science is the most outstanding fact of the modern world, and has led to the most far reaching changes in human viewpoints, in human betterment, in individual and national power. Eventually it will lead to the greatest changes in human government.

### The present position

The physics courses in the schools tended, however, to be on much too narrow a basis, favoring the boy who was going on to be a specialist rather than the boy who would become a normal citizen with an intelligent and critical interest in the subject. The courses tended to be

[2] W. H. Fyfe, "Henry Armstrong and Christ's Hospital," Sch. Sci. Rev. **19**, 1 (1937).

[3] The British Association's *Report on school certificate science* (Cambridge Univ. Press, 1928) might be of interest here.

too technical, and the correct balance between technic and information was not attained. Consequently, science masters became duly concerned and Professor Philip, in his address[4] to their association in 1930, pleaded for a "broad purpose" in the syllabus. The result was that a special committee was set up to prepare a General Science Course. Three members of this committee made a list of common things a boy should know; three others compiled a list of very important fundamental ideas and the subject matter involved in them. The two lists were compared and much was found common to both. These common data formed a basis for the new syllabus, issued in 1936. This report fostered much interest and criticism among science masters, and the general opinion was that it was unsatisfactory. Bernal[5] summed up the position by saying that the report "lacks comprehensiveness and modernity. There is a good biological section but no astronomy or geology. In the first two years of the Physics Course there is only one fact and in the third year only two facts that were discovered in the nineteenth century. In the last year there is nothing later than 1890. X-rays, wireless and electrons are not even mentioned."

In consequence of such criticism, a second report was issued containing an extended syllabus which included astronomy and geology sections. Many schools are now working along the main lines of this syllabus. In the writer's present school, for instance, one stream of pupils takes this type of course while another stream takes the more formal and specialist course concentrating mainly on physics and chemistry. Each boy, whether he is on the modern or language side, has thus done some general science when he leaves school.

It is outside the scope of this article to give the physics section of the new syllabus in detail, but a broad outline is as follows. In the first year some introductory work on light is done, together with some heat, sound, pulleys, levers, the idea of work and the efficiency of machines. This covers, approximately, 53 periods. Astronomy (motion of sun, moon and stars, phases of moon, observations of Venus and Mercury) is allotted 10 periods. Thirteen periods are devoted to the light. In the second year, 29 periods are devoted to introductory magnetism and electricity, 32 are taken up with additional properties of matter (friction, diffusion, surface tension, pumps) and 8 periods are given to a further study of astronomy. In the third year, the more mathematical parts of heat (specific, latent, mechanical equivalent) are done, and about 37 periods complete the light. The fourth year consists mainly of revision of the principles and the addition of electrical instruments, machines, motors, the telephone, etc. The report has had a good reception and doubtless will form the basis of many general science courses in the schools.

So far we have dealt mainly with the teaching in the higher grade schools. The elementary schools—those taking pupils up to the age of 14 and maintained by local boroughs or county councils—also offer the subject as part of a general science course. Several of these schools now possess excellent laboratories and workshops with a fully qualified staff. The equipment and design of these laboratories in the newer schools are often much better than in the older type of grammar school. In some of the public schools (mainly independent boarding schools), a good deal of improvement is still necessary. It is, for instance, quite possible for a boy to pursue a classics course without doing any science at all. This type of boy, in modern times, can scarcely claim to have had a general education; he can hardly place the achievements of modern science in their true perspective and cannot have a full appreciation of their true value. Again, at some of these schools, the course in physics is far too short. This is due to the lamentable fact that very little science is taught in the preparatory schools which feed the public schools. Pupils often enter via the Common Entrance Examination, which, at present, does not include science as one of its subjects. The majority of science masters would like to see the subject made compulsory in the examination. Nevertheless, the position has improved considerably in the last few years, and science has gradually filtered through the almost immovable barrier of classical tradition. Much is being done, also, to

[4] J. C. Philip, "School science—its purpose and scope," Sch. Sci. Rev. **11**, 169 (1930).

[5] J. D. Bernal, *The social function of science* (Routledge, 1939).

foster interest in the subject by work outside the classroom. Many schools have science societies and lectures on recent physical work. These societies are not confined to the upper school. The author has found that beginners can run their own society very efficiently—most of the lectures being given by the senior boys. In addition, many schools now run exhibitions, such as on Speech Days, at which experiments of a popular nature are performed. The writer has organized several of these exhibitions and has found that they are of immense value, since the pupils meet many practical difficulties which they would not normally have to face and must show a good deal of initiative in fitting up their exhibit and explaining it to an interested visitor. In this way they see that the subject has a real value in everyday life and do not treat it as just another examination trial! The writer believes that these exhibitions have a definite place and purpose in the life of the school.

**The future**

At the moment of writing the country is once again involved in war. What effect this will have upon the teaching of physics it is difficult to foresee, but already there has been much discussion about it. The two main points of current interest are (1) the effect of evacuation, and (2) the modification of the syllabus to include the applications of physics in warfare.

Physics has been one of the first subjects to suffer, owing to the removal of schools from their home towns. Most of these schools have been evacuated to the country, where the laboratory accommodation and equipment have been insufficient for the large number of pupils sent. Some schools, unavoidably, had to go to places where there was no laboratory accommodation at all, and had to remove their own apparatus to hastily improvised laboratories and workrooms. Curtailed time has forced masters to concentrate upon the essential principles, and the fitting up of the more elaborate standard experiments has had to be abandoned. In addition, a large number of evacuated schools found it necessary to dispense with the services of their laboratory assistants—a serious drawback to the teaching of the subject. Some schools, such as the writer's own, have been able to make arrangements for extra practical lessons when the laboratories are free during the session of the reception school. This has proved to be a great help.

Against all these difficulties, however, must be set some of the advantages of evacuation. Masters and boys have been able to observe how the subject is tackled in another school, have had to accommodate themselves to different types of apparatus, and have had to work in close liaison with pupils taking different examinations. Moreover, combined societies and groups can meet to discuss physical problems, and this should foster a broader interest in the subject.

A difficult problem is the question of syllabus. The examination requirements will scarcely allow much time for the inclusion of items of topical interest;[6] and yet these should be included, for the country is at war and the pupil should know the application of the subject. The general feeling seems to be that these should be discussed wherever possible, but the great danger of sacrificing the principles for general examples of wartime application must be carefully avoided. Doubtless, one of the effects of the war will be to make the subject more technical, and if this is so, our examination syllabuses will have to be made wider in order to make the teaching more elastic. At the moment there is, perhaps, a great danger of our physics teaching becoming too stereotyped. American physics teachers can, however, rest assured that the subject commands a sure place in the curriculum of the majority of our schools, and with the strong liaison which exists between the teachers and such eminent bodies as the Physical Society, the advancement of the subject should be guaranteed and its place as a vital member of our educational forces assured.

[6] See C. L. Bryant, "Applications of science in warfare," Sch. Sci. Rev. **21**, 909 (1940).

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*MAN holds within his hands the power to make the world virtually what he will.*—G. R. HARRISON, *Atoms in Action.*

# Mechanical Oscillator for Determining Moments of Inertia

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RECENTLY the writer has had occasion to add a number of laboratory experiments to a second course in physics devoted to mechanics, heat and sound. An ellipsoid of inertia experiment described by M. H. Trytten[1] seemed to deserve a place among the new experiments. The moment of inertia apparatus described by Trytten consists essentially of a heavy spiral spring with the outside end held rigid, and the inside end attached to a vertical axle having an aluminum disk mounted on its upper end. The moments of inertia of various bodies are compared by clamping each body in turn to the disk and observing the period of oscillation. During the construction of a similar device an alternative scheme was tried for obtaining the elastic restoring torque. A rigid arm was attached radially to the axle and two small helical springs were run from the end of the arm to each side of the wood frame supporting the bearings for the axle. It was found that, because of excessive damping, coupled with the necessarily small amplitudes of oscillation, accurate measurements of the period could not be made. It then occurred to the writer to run the apparatus as a self-excited oscillator by means of a powerful electric bell mechanism. Unfortunately, the period was found to be very sensitive to changes in amplitude. A realization of the electric bell paradox[2] made it evident that the driving torque should be more nearly sinusoidal. This led to a design that has proved to be very satisfactory.

Two small iron armatures were attached to the underside of the disk and arranged to move in and out of two U-shaped electromagnets excited "push-pull" by a polarized relay.[3] Figure 1 indicates the essential parts of the oscillator and the accompanying circuit. In operation, when the soft spring contactor $C$ attached to the oscillating system touches contact $A$, the voltage across $R_A$ is applied to the polarized relay. The relay is thrown to side $A$ where it remains. Magnet $A$ is thus excited, pulling the armature $A$ toward it. The pull passes through a maximum as the armature moves from a distant position into the gap of the magnet, becoming zero just as the contactor $C$ touches contact $B$. This excites magnet $B$, which attracts armature $B$, etc. The driving torque is approximately sinusoidal and 180° out of phase with that caused by friction. The period of oscillation is determined by throwing the toggle switch $S$ which essentially introduces the low resistance (0.27-ohm) impulse counter[3] $M$ in series with magnet $A$ and starts the electric stop-clock $T$. If 1000 oscillations are timed to determine the period, it is found that successive runs will agree to within 1 or 2 parts in 1000. With the present springs, the period of oscillation for no

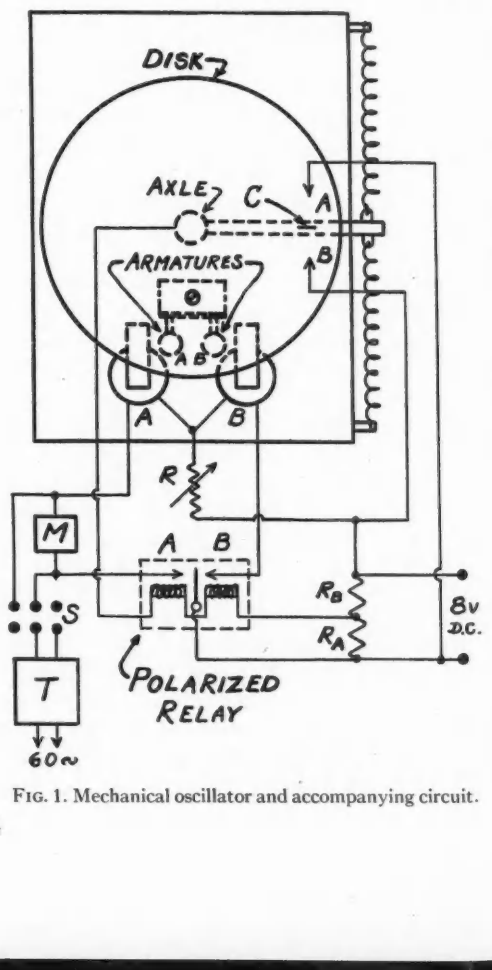


FIG. 1. Mechanical oscillator and accompanying circuit.

[1] Am. J. Phys. (Am. Phys. T.) **1**, 115 (1933).
[2] Little, Am. J. Phys. (Am. Phys. T.) **4**, 139 (1936).
[3] Gift of Western Electric Co.: Relay 215FB; message register 5H.

added mass on the disk is about 0.2 sec. The amplitude of the motion, which is controlled by the variable resistance $R$, is 10° or less, so that the restoring torque produced by the spring-and-arm arrangement does not appreciably depart from linearity. A 2-$\mu$f condenser in series with a 10-ohm resistor (not shown in the diagram) is placed across each pair of relay contacts to prevent sparking. The apparatus has been built as a self-contained unit except for the stopclock. For operation it requires from 1 to 1.5 amp at 8 v, depending on the load which the disk carries.

Students using the apparatus first determine a calibration curve by plotting for a range of values the square of the observed period against the moment of inertia of two cylindrical masses which can be screwed to the disk at known distances from the axis. The experimental points fall on a straight line with practically no scattering. Next, the test body is fastened to the disk between upright supports. If the latter are present during calibration, then one measurement of the period will determine the moment of inertia of the body about an axis coincident with the axis of the oscillator. Figure 2 illustrates a section through an ellipsoid of inertia as obtained by one group of students for an unsymmetrical test body. The ellipse was drawn through the experimental points by the familiar pin-and-string method. As part of the experiment, a similar run is made on a right cylinder for which a computed ellipse is plotted before the experimental points are determined.

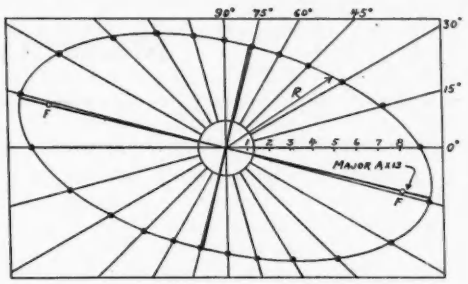


FIG. 2. Experimentally determined section through an ellipsoid of inertia; $R = I^{-\frac{1}{2}} \times 10^3$ gm$^{-\frac{1}{2}}$ cm$^{-1}$.

It is believed that the added complexity of the present device as compared with the beautiful simplicity of the one described by Trytten is not without its recompense. The gadget runs by itself, thereby arousing curiosity in the mind of the student and challenging him to explain its operation. Inevitably, certain fundamental features of an oscillating system will be brought out in a direct way which can be readily visualized and later carried over to the corresponding electric case. This, of course, is in addition to the fact that the device does an excellent job of comparing moments of inertia. It is efficient with regard to laboratory time, for calculations can be made while additional data are being secured. Either the present apparatus or the one described by Trytten can be used for moment of inertia experiments of the more conventional sort usually performed with a torsion pendulum.

## Appointment Service

REPRESENTATIVES of departments or of institutions having vacancies are urged to write to the Editor, Columbia University, for additional information concerning the physicists whose announcements appear here or in previous issues. *The existence of a vacancy will not be divulged to anyone without the permission of the institution concerned.*

### POSITIONS WANTED

**31.** Ph.D., Columbia. Years of experience as head of departments of physics in colleges and universities. Author of new type of laboratory manual. Designer of many new types of simplified apparatus. Research in radio, acoustics and methods of teaching physics.

**33.** M.S., experimental physics, coupled with thorough background of courses in professional education. Has taught physics and mathematics for 3 yrs in large high school. Desires position as instructor in high school physics in a university or college experimental or training school.

**34.** Ph.D., M.S., Penn State. Age 40, married. 15 yrs teaching experience in colleges and universities; 5 yrs head of department in small college; industrial research experience. Interested in teaching, research and administrative work in a small college.

**35.** Ph.D., Purdue; M.A., British Columbia. Age 27, married. Experience: 5 yrs university teaching; 2 yrs secondary school teaching; 5 yrs research in analysis of liquids by x-rays. Interested in teaching and research.

**37.** Ph.D., engineering physics, mathematics and physical chemistry, Illinois; A.B., education. Age 37, married. University, teachers college, junior college and high school experience; 1 yr editorial work. Especially qualified for survey courses.

**38.** M.S., physics, Lehigh. Age 35, married. 10 yrs teaching college and university, particularly laboratory instruction. Interested in college teaching, or in industrial production and developmental work in precision instruments.

**39.** Ph.D., physics, large Eastern university. Age 45, married. 16 yrs university and college teaching experience, including 9 yrs head of physics department of a small liberal arts college. Research in optics and spectroscopy. Desires opportunity for research in teaching or industrial fields.

### VACANCIES

**1.** Experimental physicist for industrial research foundation, Ph.D. preferred. Must be capable of designing and constructing apparatus and able to cooperate with other research workers. Knowledge of x-ray diffraction and high vacuum technics essential.

**2.** Experimental physicist with experience in using spectrographic and general optical equipment for research.

**3.** An Eastern college needs 2 instructors for second semester only. Probable salary, $1100 for semester. One appointment may become permanent at salary to be determined by qualifications of individual.

# NOTES AND DISCUSSION

## An Improved Young's Modulus Apparatus

IN a complete study of the factors involved in Young's modulus, it is necessary to obtain readings on at least three wires. Ordinarily the student, or his instructor, must change wires on the apparatus during the course of the experiment. Since this is an unnecessary, time-consuming process, the three selected wires were permanently attached to the Young's Modulus apparatus built for this laboratory.

Figure 1 illustrates the essential features of the design. The three wires, spaced $\frac{3}{8}$ in. apart, are securely fastened in place by means of a loop around a hardened screw and then clamped between the hardened steel back plate and heel to prevent creepage. The accuracy of wire-length measurement is improved by clamping the wires at the bottom of the plate.

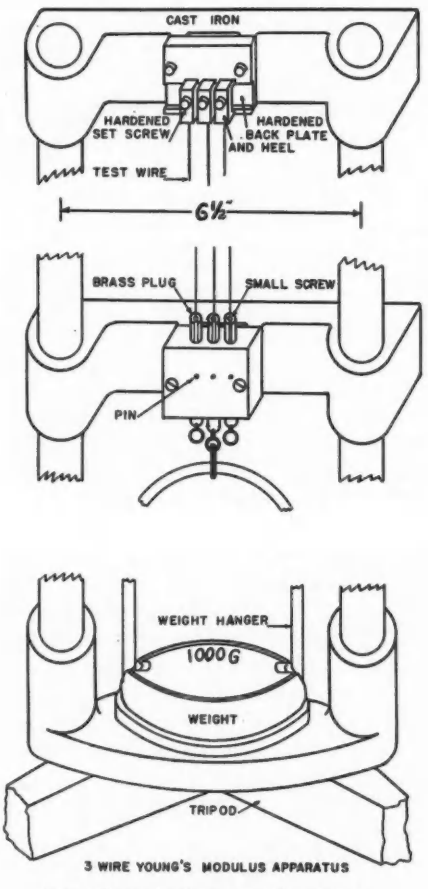


FIG. 1. Three-wire Young's modulus apparatus.

The lower end of each wire is securely held in a brass plug by a small screw. Each plug has a groove cut half its length so that a pin set into the block through which the plug slides limits the upward motion of the plug and prevents twisting of the wire. This holds the index line in correct position for measurement of change of length by any of the usual methods.

To avoid the annoyance of falling weights, the weight hanger and weights are built as shown in the diagram. The weights cannot fall from the hanger, but may be removed easily by turning the plane of the weights through an angle of about 30°.

The author wishes to express his appreciation to Professor C. J. Overbeck and Mr. C. H. Miller of this laboratory.

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## A Thermodynamic Paradox

OCCASIONALLY a question is sent to a physics department that raises interesting points of theory. The following paradox was recently sent in by the captain of a Pacific freighter who was puzzled by the apparent contradiction to the second law of thermodynamics mentioned below. While it is probably not new, it has several points of interest to teachers of physics. Imagine two long vertical tubes $A$ and $B$ (Fig. 1) connected at $D$ and filled with liquid of density $\rho$. If by means of two heat reservoirs one is maintained at a temperature $T'=T+t$, while the other, of height $h$, is kept at $T$, the liquid in one will stand at a higher level than in the other. If the expansion coefficient $(1/v)dv/dT$ is $\alpha$, where $\rho=1/v$, the difference in level will be $\alpha ht$. If we allow the liquid to flow over the top of $A$ and do work, we would obtain $g\alpha th$ ergs for every gram that passes from $A$ to $B$. To maintain the temperatures constant, the reservoir at $T$ must absorb the heat $Ct$, where $C$ is the specific heat, and as a gram of water flows back from $B$ to $A$ through $D$ an amount of heat must be taken from the second reservoir also equal to $Ct$. We thus have an apparent contradiction to the first law of thermodynamics. Students may at first try to avoid the difficulty by bringing in the dependence of the specific heat on the pressure. The specific heat $C'$ at $D$ must be larger so that $C'-C=\alpha gh$ in order to agree with the first law. There are two objections to this. First, the efficiency of the heat engine, $W/Q$, would be $\alpha gh/(C+\alpha gh)$, and, by taking long tubes (and for ordinary substances they would have to be excessively long), this ratio may be made to approach unity as closely as we wish, in contradiction to the second law of thermodynamics. A second objection to this explanation is that while the specific heat may increase with pressure, the proportionality factor is not $\alpha$ or $(1/v)dv/dt$, but the second derivative. The change with pressure is given

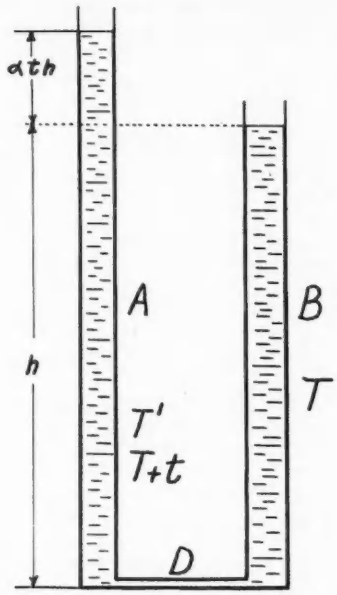


FIG. 1. Work is obtained by liquid falling from the top of tube $A$ through the difference in height $\alpha th$ into tube $B$. The difference in height is maintained by two reservoirs that keep tube $B$ at temperature $T$, and $A$ at a higher temperature $T'=T+t$.

by the formula $(\partial C/\partial p)_T=-T(\partial^2 v/\partial T^2)_p$, and a substitution of the values for mercury, for example, shows that the actual change in $C$, in place of being $\alpha gh$, is a hundred times too small.

We may avoid these difficulties if we take account of the fact that the heats, $Q_1$ put in from the reservoir at $T'=T+t$, and $Q_2$ given up to the reservoir at $T$, include also the heats due to change of pressure on the gram of liquid that moves in the circuit. As the heat of compression is given by $(\partial Q/\partial p)_T=-T(\partial v/\partial T)_p$, and the total pressure difference is $\rho hg$, the heat that must be taken out by the reservoir at $T$ to keep the temperature constant is $T(\partial v/\partial T)_p\rho hg$. Thus

$$Q_1=C't+T'(\partial v/\partial T)'\rho hg,$$
$$Q_2=Ct+T(\partial v/\partial T)\rho hg.$$

Since

$$C'=C-T(\partial^2 v/\partial T^2)\rho hg$$

and

$$T'\left(\frac{\partial v}{\partial T}\right)'-T\left(\frac{\partial v}{\partial T}\right)=(T'-T)\frac{\partial}{\partial T}\left(T\frac{\partial v}{\partial T}\right)$$
$$=(T'-T)\frac{\partial v}{\partial T}+(T'-T)T(\partial^2 v/\partial T^2),$$

the difference $Q_1-Q_2$ from the foregoing equations reduces to $(\partial v/\partial T)hg\rho(T'-T)$, and this is equal to $\alpha ght$, the work gained, as required by the first law. The efficiency of the process $W/Q_1$ is $\alpha gh(T'-T)/(Ct+\alpha ghT')$, and this is always less than $(T'-T)/T'$, as required by the second law.

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## The Significance of Curl

When a student first meets the concept of "curl" he sometimes finds that the ideas back of the equations remain for a time rather hazy. The following treatment is probably not new, but I think is certainly not well known. I present it in the hope that it may prove helpful.[1]

*In Cartesian Coordinates.*—The customary expression for the $z$ component of the curl of a vector field is

$$\frac{\partial Y}{\partial x}-\frac{\partial X}{\partial y}, \tag{1}$$

where $X$ and $Y$ stand for the $x$ and $y$ components of the field at the point in question. If the $Z$ axis is also the axis of curl this expression represents the entire curl. Let this be the case, and let us consider a small region in the neighborhood of the $X$ axis.

First let $Y$ be independent of $x$, so that the curl reduces to the second term of (1). Then, if the curl is positive and $X$ is small, (1) shows that, in the neighborhood of the $X$ axis, the lines of the field have somewhat the form sketched in Fig. 1(*a*)—with the field *curving* toward the left.

Next let $X$ be independent of $y$, so that the curl is given by the first term of (1). Then, if the curl is positive, and $X$ is small, the lines of the field in the neighborhood of the $X$ axis are somewhat as shown in Fig. 1(*b*).

Both figures show effects that suggest a *curling* of the field in the positive sense around the $z$ axis.

*In Cylindrical Coordinates.*—As before let the $Z$ axis lie along the axis of curl. The field at any point in the neighborhood in question may be resolved into a radial component $R$ and a transverse component $T$. Then either the use of Stokes' theorem or a straightforward transformation from Cartesian to polar coordinates $r, \theta$ puts the expression for the curl into the form

$$\frac{T}{r}+\frac{\partial T}{\partial r}-\frac{1}{r}\frac{\partial R}{\partial \theta}. \tag{2}$$

If the field is entirely radial and is symmetric about the axis, (2) shows that the curl vanishes. Thus the gravitational field of a single particle has no curl, and the same is

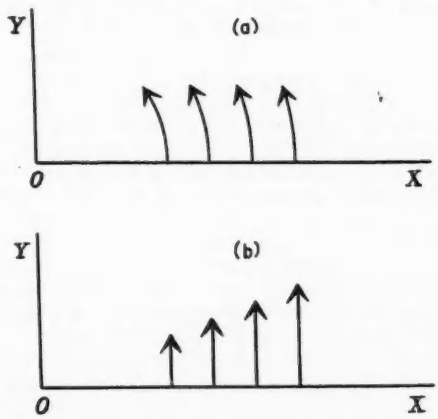


FIG. 1. Two contributions to curl. The axes are right-handed.

true of the field of an electrically charged conducting sphere at rest, the field of a small magnet pole at rest, and the velocity field in a body of fluid where the flow is radially inward or outward and is symmetrical about the center.

If the field has no radial component the last term in (2) vanishes, and the field cannot be irrotational unless the sum of the first two terms vanishes. The second term contributes to the curl in a manner similar to that shown in Fig. 1(*b*). If this second term vanishes, and the field continues to be entirely transverse, the lines of the field must be similar to those in Fig. 1(*a*), so that the contribution which $T/r$ makes to the curl arises from the *curving* of the field.

If the sum of the first two terms vanishes, and the field is still entirely transverse, we can integrate the sum and find that $T$ must be inversely proportional to $r$. If $T$ decreases with increasing $r$, and does so more rapidly than the inverse first power, the second term in (2) is the more important and the curl has one sense. If $T$ increases with $r$, or decreases less rapidly than the inverse first power, the curl has the opposite sense.

A case in which $T$ is inversely proportional to $r$ is that of the magnetic field which accompanies a steady electric current in a long straight wire, and it is well known that this field is irrotational.

As another example of (2) consider the case of a rigid disk that spins with angular velocity $\omega$ about an axis perpendicular to the plane of the disk. The linear velocity of a particle at a distance $r$ from the axis is $\omega r$, and the field of this linear velocity is entirely transverse. If we set $T=\omega r$ we see that the first term in (2) contributes $\omega$ to the curl—corresponding to a curving of the path—and that the second term contributes another $\omega$—corresponding to the greater speed at greater distance from the axis. Thus the reason for the factor 2 in the well-known expression for the curl of the linear velocity becomes evident.

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[1] See also V. Karapetoff, Phil. Mag. **31**, 528 (1916); Am. Math. Monthly **35**, 14 (1928).

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## Mathematics and Interpretation

IN my few contacts with the new quantum and wave mechanics, one very striking phenomenon has made itself evident. Never has there failed to be reference to the bundle of energy, the *quantum*, and the particle of electricity, the *electron*. Yet at the same time there is also mentioned the fact that both the light quantum and the electron have wave properties; the evidence being that both light and electron diffraction patterns have been obtained.

Everyone is familiar with these concepts. However, one troublesome point appears. This is the attempt to visualize physically what one calls a light quantum or an electron in terms of what one observes in his ordinary everyday experience. Frequently, physicists regard the electron as a negatively charged particle and teach this concept. In the same way, they regard a quantum of light energy as a little bundle, equivalent to a particle. On the other hand, the diffraction experiments make it permissible for the quantum and electron to be considered as waves.

Heretofore, the wave and particle properties of physical entities have been considered as mutually exclusive. Never in one's previous experience has there been encountered a physical entity that demanded consideration as a wave as well as a particle. The question that naturally arises is: How can this be? This is a perfectly legitimate question that cannot be answered by any attempt to visualize the situation physically.

One can say only this: In dealing with physical phenomena involving light or the electron beam, one obtains certain experimental results that cannot be explained by a pure wave theory or a pure corpuscular theory. This fact immediately eliminates the possible physical visualization of such phenomena, since assigning this dual property of wave-particle nature to a physical quantity is not contained within one's physical experience. So what does one do? He postulates a mathematical theory which, when properly manipulated and interpreted, will predict the experimental results. And here lies the crux of the situation. Since it is more comforting to use some word or words to express the physical significance of the equations, the words *electron* and *quantum* have been chosen. An attempt to interpret these words on the basis of the apparent discontinuous emission of what we call a quantum or an electron seems to be a reasonable excuse for immediately regarding them as particles. But with what justification? None whatsoever, except to invent a physical picture. But must a mathematical symbol have a physical meaning? I think not. It can be interpreted in many lights, but a physical picture cannot necessarily be extracted from it. One can use only those definite properties he has found to construct an equation corresponding to them and then he can substitute the physical measurements that he has made into this equation. In this case there is no question at all about a physical interpretation since the original equation is constructed on that basis.

However, equations in general and quantum- and wave-mechanical equations in particular are not so fortunate. Generally, equations admit one of two alternatives: Either these physical phenomena can be pictured, or the symbolisms in the equations, at least for the present, are not adapted to physical interpretation in terms of what one can conceive from past experience. The latter is true about the wave- and quantum-mechanical equations.

Mathematical symbols have been innocent victims of misrepresentation, and the agents who misrepresented them have, for the most part, been teachers. In the attempt to obtain a plausible concept, the symbols have been given properties which they never claimed to possess, because of the mistaken notion that the prerequisite for the existence of mathematical symbols is their interpretation in terms of physical entities. Mathematical symbols are only conveniences which the human mind can manipulate in order to come to conclusions coinciding with experimental evidence.

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### Report of the Committee Representing the Association Before the American Council on Education

THE committee of the American Association of Physics Teachers attending the meeting of the American Council on Education, May 2–4, 1940, consisted of C. J. Lapp, H. B. Lemon and R. M. Sutton. The reflections of the committee on the sessions and conferences attended are expressed in the following notes prepared by its members.

Many of the problems which the Council faces are enormous and fundamental to the educational program of the country as a whole and involve in many cases the basic philosophy of education, on which there is by no means unanimity. Physics must see itself in correct perspective in relation to the problems of general education, and it is doubtful whether most teachers of college physics have any such perspective.

There is a great need for cooperation and understanding between educational groups, and more recognition of the honesty and ability of those who are working in other fields. "We are victims of lack of confidence in one another."

The problems of the Council seem to center chiefly about general education at the elementary and secondary school level, where, it is obvious, the bulk of all educational funds is being spent and where the results are most important to the largest number of people. There is so little emphasis upon the problems of college education and so little concern over the status of specific subject-matter fields in secondary education that no immediate niche is evident for a society such as the American Association of Physics Teachers. If we are to be effective, along with other groups interested primarily in the problems of a single field, we must carve our own niche. This will mean a willingness to cooperate with others and to subordinate the grinding of our own axes in the interest of the success of large-scale programs such as, for instance, the improvement of the position which the sciences in general may rightfully be expected to hold in the whole picture of secondary school education. It appears that we may progress faster at first with our own committees on cooperation with the other sciences, but that as their findings become fuller and the problems become more sharply delineated, we may then reasonably expect to push them ahead with the Council. There are great possibilities in the joint action of our committees and our American Council representatives. The former may well supply the latter with effective ammunition at first, and may assist in formulating a program which we, and others, may lay before the Council with some expectation of further progress.

The Association may well bewail the deplorable condition of physics and other sciences in secondary education, but to do anything effective about the problem, we must be willing: (1) to see the problem in correct perspective; (2) to cooperate with whatever secondary school groups we can. This means that we must forsake some of the Brahmin attitudes which we have developed in the past. We must realize that few secondary schools can support a full-time person in physics alone, and that our problem is in large measure one of finding better means of helping those teachers now on the job who must teach physics along with other subjects. These teachers are not specialists in physics and we cannot make them specialists, but we should be able to do something that will help them where they are.

One of the major problems, from the standpoint of physicists, seems to be to stem the tide toward social sciences and stimulate more attention upon the physical sciences in the schools. This probably means that we must seek new lines of approach in presenting physics to the public so that it will recognize the desirability of the subject.

Judging from the discussion of the work of the Commission on Teacher Education, the emphasis is upon broad general culture and there seems to be little interest in the training of specialists as teachers. One speaker said in so many words that the time has gone when the schools can use specialists in a field. We must either challenge this attitude or learn how to cooperate with it effectively. Most college teachers are specialists and many are primarily interested in the training of new specialists. Physicists have been loath to pay sufficient attention to the scientific needs of the nonspecialist and have already alienated many who should know more about physics than they do. Because of our intense interest in our own field, we have been slowly cutting our own throats. We may still need an "aristocracy of brains," but we must not overlook our responsibilities and our opportunities in training nonspecialists.

The committee had an extended conference with the Director of the Commission on Teacher Training with the following results:

1. He agreed to call a conference of men in all the college subject-matter science fields to meet with administrators and subject-matter people from the secondary schools to talk over the common problems of teacher training and curriculum building in order that the whole complex problem may be more intelligently comprehended and attacked.

2. He recognized that it is important that the numerous committees representing various organizations have their work coordinated and expressed willingness to have his department act as such a coordinating agency.

Because of the character of the Council and its broad base of operation, there naturally was considerable talk about democracy and the social order, and a concern over the ills of the social order. Any physicist must realize that his science has far-reaching effects upon society, but few ever do anything effective about it. We are perhaps much too subject-centered and we may have to learn how to cooperate better before we can do much for ourselves or for education generally.

For the most part, it seemed clear that the members of the Council who spoke at the various sessions were honestly trying to get somewhere with enormous problems of general education, problems to which we as an organization have given practically no attention. It is still a question as to how much we can do by pitching in on some of the broader problems, but we may well work out a plan of action that should have some promise of success within a limited area. Although the power to vote in the Council may some day be more important than it appears to be at present, it will mean little until we have a program of action which we wish to push. More can be expected if we succeed in arousing the cooperation of our sister sciences.

We probably have in the past looked too much upon secondary school physics primarily from the standpoint of preparation for college physics. We should lend every

effort to make certain that school physics meets the needs of the bulk of students who will probably never see college. There are enormous difficulties in the way of doing that when one considers that $5\times10^5$ out of $7\times10^6$ students are still in secondary schools with enrolment less than 200.

The Council is a working body struggling valiantly with the tremendously complex problems facing education in America today. The men in charge of the individual programs being sponsored by the Council are extraordinarily able men working vigorously and intelligently at their jobs. It has been clear to some of us for many years that American education cannot fulfill its destiny unless some way is found to educate for character as well as to impart facts and statistics. Evidently the best leadership in the country at present also recognizes this important fact and is endeavoring to find ways and means of injecting character education into the program of general education.

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**Shall the Association Establish the Grades of Fellow and of Member Emeritus?**

A COMMITTEE of the American Association of Physics Teachers consisting of A. G. Worthing, *Chairman*, Louise S. McDowell, Duane Roller and L. W. Taylor has been appointed to consider the matter of establishing membership grades of *fellow* and of *member emeritus*. The committee members have rather definite personal views, but believe that, on questions so important as these, a much wider expression of opinion should be available before definite proposals are offered to the Association at the annual meeting in Philadelphia. Interested members of the Association are therefore urged to communicate their views on these questions to the chairman, Professor Worthing, University of Pittsburgh, *not later than December 15*.

THE GRADE OF FELLOW

The proposal that the Association establish a membership grade of *fellow* was made by Professor G. W. Stewart in a statement which read, in part, as follows:

> Since the Association recognizes the great need for appropriate emphasis upon the teaching of physics, it should honor the members who carry on creative work in this field, especially work of permanent value. The standards of selection should be at least as high as those set up for the grade of *fellow* by the American Physical Society. The award should be based upon *continued* creativeness in, for example, the development of new experiments, apparatus and materials for instruction, such new methods of instruction as win favor, and researches into the effectiveness of different types of instruction. The Association should not be afraid of getting into a desert of educational quibbling. It should rather expect to be its own best physician. The reason is simple. Different subjects and aspects of physics require different methods of approach that are inherent in the peculiar abstractness of the subject or aspect concerned. A physicist can appreciate and get into this kind of detail.

Among the arguments that may be offered in favor of establishing the grade of fellow are:

(1) Distinguished teaching and outstanding contributions to teaching deserve recognition to a much broader extent than is possible through the medium of the Oersted Medal awards.

(2) When a teaching position is to be filled, able, creative teachers who have faithfully devoted their energy and time to instruction and the improvement of instruction, but who lack public recognition either for research or for teaching, are often at a disadvantage in comparison with those who merely have research recognition as shown by the grade of fellow in another society.

Arguments offered in opposition to the grade of *fellow* include:

(1) Class distinctions are undesirable in a democratic organization.

(2) Recognition often would be given where not deserved and not given where deserved, because the real evidences of good teaching and contributions to teaching are often highly localized. Since, for example, ingenuity in devising new experiments does not necessarily imply good or distinguished teaching, and published papers cannot justly serve as a sufficient criterion, the difficulties of selection would seem to be unduly great.

*An alternative proposal.*—Instead of having the grade of *fellow*, citations for distinguished teaching or significant contributions to instruction should be awarded annually to a relatively small number of teachers, say, 5 to 10. These citations should be published in the *American Journal of Physics*, with detailed accounts of the basis for the awards. This proposal seems to be subject generally to the same favorable and unfavorable arguments as those advanced for the establishment of the grade of fellow.

THE GRADE OF MEMBER EMERITUS

Arguments in favor of the establishment of the grade of *member emeritus* include:

(1) Retention of identity with a society and its members after retirement from active professional duty is a privilege that would be highly prized by retiring members.

(2) Because of their experience in teaching and the fact that after retirement there is usually more time available for such activities, members emeritus can make many contributions to the cause of teaching through the publication of articles and participation in Association activities.

Arguments advanced against the establishment of the grade of *member emeritus* include:

(1) The maintenance of this grade of membership may add to the financial burden of the Association.

(2) The welfare of the Association demands that there be greater emphasis on "new blood" (The committee is not at all certain that this is a valid argument).

If this grade of membership is established, questions such as the following must be settled:

(*a*) How long should one have been a member of the Association to be eligible for election upon retirement to the grade of *member emeritus?* (*b*) Should the election be automatic, or upon request of the retiring member? (*c*) Should members emeritus pay regular dues, the same dues as, say, junior members, or no dues? (*d*) Should they have the privilege of voting, of holding office, of receiving the journal?

# Available Graduate Appointments and Facilities for Advanced Study in Physics

This fourth survey of opportunities for advanced study in physics is based on information obtained from a questionary sent to all departments of physics and organizations in the United States and Canada known to offer graduate work or other facilities for advanced study. The information must be interpreted as estimates rather than exact descriptions of the conditions that will prevail in these departments next year.

Few appointments are available compared with the probable number of candidates and the mere publication of the list obviously does not improve this situation. The purpose of the information is to enable the student to choose intelligently the institutions in which he desires and can afford to study, and hence to render it unnecessary for him to send out applications promiscuously or to depend entirely on information obtained casually or through connections of a personal nature. If the list is used judiciously by students and their advisers, its publication will tend to prevent rather than encourage an unnecessarily large number of applications. *The student, with the help of his adviser, should select a small number of institutions in which he feels best qualified by his interests and abilities to study, and in any one of which he would be willing to accept an appointment as soon as it is offered. When he accepts an appointment, other pending applications should be withdrawn immediately.*

**Note**—Unless otherwise specified, the application must be filed with the person designated *before March 1*; if no person is designated, address the head of the Department of Physics. "M only" means that the master's degree is the highest degree granted. "No form" means that a special application form is neither furnished nor required. The numerals preceding the parentheses indicate the probable number of vacancies for next year; those in parentheses indicate the total number of appointments existing in the department; e.g., 2(3) *fellows* means that three fellowships exist, two of which probably will be vacant and open to competition in 1941–1942. Except where otherwise indicated, appointees do not have to pay tuition (t.) or fees (f.). Under *R* are listed fields of research that are stressed in the department and under *U*, unusual facilities for research and study.

**A. and M. College of Texas,** Dean T. D. Brooks, College Station, Tex. M only. 2(4) *graduate assistants*, lab. teach., $405. Residents of state pay $50 t.; others, the nonresident t. of their state univ. *R:* spectroscopy; x-ray diffraction; polarimetry; electronics.

**Amherst College,** Prof. S. R. Williams, Amherst, Mass. M only. 2 *assistants*, study and research, $900. *R:* elastic properties of rubber; correlations between hardness and magnetic properties.

**Baylor University,** Prof. S. R. Spencer, Waco, Tex. M only; no form. 3(3) *graduate assistants*, lab. asst., 15 hr/wk, $300 to $400 less t. and f.

**Boston College,** Chestnut Hill, Mass. May 1; M only; no form. 2(4) *graduate assistants*, 8 hr/wk general lab., $300. Appointees must have had 2 yrs advanced physics, advanced calculus, and differential equations. *R:* electronics; electromagnetic waves. *U:* electronics.

**Boston University,** Dean H. M. LeSourd, 84 Exeter St., Boston, Mass. Apply any time. 1(1) *fellow*, $600 and tuition; 3(3) *assistants*, tuition, 3(3) *scholars*, $200 less t. All appointees pay about $15 f.; service consists of lab. asst., etc. *R:* spectroscopy; many fields in mathematical physics. *U:* 30-ft Littrow spectrograph, Lummer-Gehrcke plates, echelons, étalons.

**Brown University,** Registrar of Graduate School, Providence, R. I. 1–2(1–2) *scholars*, study and research, t. and f. remitted; 2(6) *graduate assistants*, 18 hr/wk lab. teach., $550 first year; 2(3) *minor assistants*, 6 hr/wk service, t. and f. remitted; 1–2(1–2) *fellows*, study and research, $500 less $300 t., unless remitted by summer research or other service. *R:* properties of single metal crystals (electron diffraction, photoelectricity, contact potentials, conductivity, magnetic susceptibility); theoretical and experimental acoustics, including supersonics; atomic theory calculations (e.g., atomic wave functions); foundations of physics. *U:* single metal crystal technic; electroacoustics.

**California Institute of Technology,** Graduate Dean, Pasadena, Calif. Feb. 15. 8(25) *graduate assistants*, lab. and lec. teach., room, board, t. and f. *U:* nuclear physics; cosmic rays; spectroscopy; physics of solids; x-rays; ultra-short radio; low temperatures; geophysics; biophysics; isotopes.

**Carnegie Institute of Technology,** Chairman of Admissions, Pittsburgh, Pa. 2(7) *teaching assistants*, lab. and other teach., $800 first year, $1000 for reappointments. Appointees pay $405 t. and f. *R:* electricity and magnetism; vacuum tube circuits; optics; nuclear and molecular physics. *U:* molecular beams; infra-red spectroscopy; thermomagnetic and galvanomagnetic effects; optical properties of metals and thin films; low temperatures.

**Case School of Applied Science,** Prof. R. S. Shankland, Cleveland, O. M only. 1(4) *graduate assistants*, lab. and lec. asst., $600. *R:* acoustics; electron diffraction; ultra-violet spectroscopy; x-ray diffraction; meteorology. *U:* acoustics; electron diffraction.

**Catholic University of America,** Dr. R. J. Deferrari, Secretary General, Washington, D. C. 2(2) *K. C. Scholars*, no service, board, room and t., less lab. f. *R:* theoretical physics; ultrasonics; spectroscopy.

**Central Scientific Company.** Dr. M. N. States, Chicago, Ill. Apr. 1; no degree awarded. 3(3) *industrial fellows*, development lab., factory inspection, etc., $1200 for 12 mo, less deductions for social security, etc.

**Charles A. Coffin Foundation.** The Secretary, Schenectady, N. Y. Jan. 15. *Charles A. Coffin fellowships* for research in electricity, physics, and physical chemistry either here or abroad; or some portion or all of the fund may be used to further research at any institution in this country.

**Colorado College,** Prof. P. E. Boucher, Colorado Springs, Colo, May 1; M only. 1(1) *graduate teaching fellow*, 6–10 hr/wk lab. asst., remission of $225 t. *R:* x-rays; high frequency measurements; optics; photography.

**Columbia University,** New York, N. Y. 5(15) *assistants*, lab. and other teach., $1000 less $20 f.; 3(4) *part-time assistants*, lab. asst., grading, $200 to $500 less course fees; 2(2) *fellows*, study and research, $1000 to $1500 less $400 (or less) t.; 2(2–4) *scholars*, $400 to $500 less $400 t. For assistantships apply by letter before Mar. 1, and for part-time assistantship before July 1, to Prof. George B. Pegram; for fellowships and scholarships, obtain form from The Secretary of the University. *U:* molecular beams; magnetism; properties of solids; cyclotron; nuclear physics.

**Cornell University,** Prof. R. C. Gibbs, Ithaca, N. Y. Preferably before Feb. 10. 4–7(30) *graduate assistants* [teaching (26), research (4)], 1(2) *fellows*, $600 less $40 f. *R:* nuclear physics; x-rays; electronics and ionics; x-ray and electron diffraction; spectroscopy (visible and ultraviolet); cosmic rays; quantum mechanics (radiation, nuclei and solids).

**Dartmouth College,** Prof. Norman E. Gilbert, Hanover, N.H. Apply any time; M only; no form. 2(4) *graduate assistants*, half-time lab. and lec. asst., $800.

**De Pauw University,** Prof. Orrin H. Smith, Greencastle, Ind. M only; Apr. 1 or later; no form. 0(1) *graduate assistant*, lab. asst., $200 or more. *R:* x-rays; ionization in gases.

**Duke University,** Dean Calvin Hoover, Durham, N. C. 2(3) *fellows*, $600 to $700 less $281 t. and f.; 3(4) *graduate assistants*, $500 to $550 less $241 t. and f.; 1(2) *scholar*, $400 less $281 t. and f. Services for all appointments consist of lab. asst. and grading. *U:* atomic and nuclear physics; magnetism; spectroscopy.

**Harvard University,** Cambridge, Mass. 8(14) *teaching fellows* and *half-time instructors*, teach., $1000 less $200 t.; 2(5) *half-time assistants*, grading problems, $100 to $300 less $400 t.; several *research assistants*, not usually available to new students; 2(8) *fellows* and *scholars*, $400 to $900 less $400 t. For fellowships and scholarships apply on form before Mar. 1 to Graduate Dean, School of Arts and Sciences; for other appointments apply by letter immediately to Department Chairman. *U:* high pressures; electric oscillations; spectroscopy; cosmic rays; mass spectrography; ionosphere studies; nuclear physics; acoustics.

**Haverford College,** Haverford, Pa. M only. No graduate appointments available for 1941–42.

**Indiana University,** Prof. Allan C. G. Mitchell, Bloomington, Ind. 3–5(6–7) *graduate assistants*, 8 hr/wk teach., $500 to $700. *R:* nuclear physics; theoretical physics. *U:* cyclotron.

**Iowa State College,** Graduate College, Ames, Ia. 2(7) *graduate assistants*, class and lab. asst., $540; 1(2) *teaching fellows*, lab. asst., $450. All appointees pay $12 f./quarter. *R:* applied physics. *U:* electronics; physics of crystals; cathode rays; biophysics; physics of soils; photography; illumination.

**Johns Hopkins University,** Baltimore, Md. 2(6) *junior instructors*, 2 hr/wk conference, 3 periods/wk lab. asst., $750 less $336 t. and f.; 6 *laboratory assistants*, 2 periods/wk asst., remission of $336 t. and f.; 5(5) *university scholars*, remission of $300 t. but not $36 f. For scholarships apply on form supplied by Registrar's Office; for other appointments, by letter to Prof. J. C. Hubbard. *R:* optics; vacuum, visible, and infra-red spectroscopy; atomic and molecular structure; x-rays; nuclear physics; supersonics; theoretical physics.

**Lafayette College,** Prof. C. McC. Gordon, Easton, Pa. M only; no form. 1(2) *assistants*, lab. and lec. asst., grading, $500. *R:* electronics; radio; short waves; mathematical physics.

**Louisiana State University.** Prof. D. V. Guthrie, University, La. M only. 4(10) *graduate fellows*, lab. teach., $450 less $60 t. and f. *R:* x-rays; electronics; spectroscopy.

**McGill University,** Prof. A. Norman Shaw, Montreal, Can. 5–6 *demonstrators*, 12 hr/wk service, $600 to $700; 2–3 *half-time demonstrators*, 6–7 hr/wk service, $300 to $350. *R:* spectroscopy; electric waves; geophysics.

**Massachusetts Institute of Technology,** Prof J. C. Slater, Cambridge, Mass. 4–6(18) *teaching fellows*, teach. or lab. asst., $500 to $600 and t.; several *tuition scholars*, study and research, t. and f. of $600 remitted; several *half-tuition scholars*, study and research, half of t. and f. of $600 remitted. *U:* spectroscopy; electronics; x-ray crystal structure; high voltage, nuclear structure, and radioactivity; applied optics; classical and modern theoretical physics.

**Michigan State College,** East Lansing, Mich. No graduate appointments available. *R:* x-rays; spectroscopy; geophysics; supersonics.

**Mount Holyoke College,** South Hadley, Mass. M only. 1 *fellow*, study, apply on form, $500 and remission of t. and f.; 1 *graduate assistant*, 10 hr/wk general asst., apply by letter, t. and f. remitted. *R:* effect of stress on electrical resistance; discharge through gases; Geiger counter circuits and cosmic rays.

**New York University,** Washington Square College, Prof. C. W. Van der Merwe, New York, N. Y. March 31. 2(3) *graduate assistants*, $1000; 0(2) *instructors*, $1800. Appointees pay $3 to $6 f.; teach 12 hr/wk. *R:* theoretical and experimental nuclear physics; band, infra-red and hyperfine spectroscopy.

**Northwestern University,** Graduate Dean, Evanston, Ill. ?(1) *fellow*, study and research, $500–$800; 1(1) *tutorial fellow*, 8 hr/wk tutoring, board, room and t.; 5(5) *graduate assistants*, lab. asst., 12 hr/wk, $500–$800+½ t. *R:* photoelectricity; spectroscopy; supersonics; electric discharge; low temperature; x-rays. *U:* new laboratory and equipment in Technological Institute.

**National Research Council,** 2101 Constitution Ave., Washington, D. C. Address communications to the Secretary, Fellowship Board in the Natural Sciences.

**Ohio State University,** Columbus, O. 10(22) *graduate assistants*, lab. teach., $450; 1(1) *assistant*, research asst., $990; 1–2 (varies) *fellows*, study and research, $500; 1–2 (varies) *scholars*, study and research, $300. For all assistantships apply by letter to department chairman; for other appointments apply on form to Graduate Dean. *R:* nuclear physics; infra-red spectroscopy; band spectra; x-rays and crystal structure; theoretical physics; Zeeman effect. *U:* nuclear physics; infra-red spectroscopy; Zeeman effect.

**Oklahoma Agricultural and Mechanical College,** Stillwater, Okla. M only. 1(2) *graduate assistants*, teach quiz or lab. sections, $450.

**Oregon State College,** Prof. W. Weniger, Corvallis, Ore. 1–2(4) *graduate assistants*, teach. and lab. asst., $540 less $32/term t. and f. 1(1) *fellow*, $750 less $32/term t.

**Pennsylvania State College,** Prof. W. R. Ham, State College, Pa. Jan. 29. *Graduate assistants*, preference given to applicants with M degree, half-time teach., $700; *graduate stipend scholars*, 6 hr/wk teach. asst., $300; *graduate scholars*, 4 hr/wk teach. asst., remission of t. and f. All applicants not having M degree must take competitive examination. *U:* molecular beams; spectroscopy; supersonics; diffusion of gases in metals; physics of high pressures; specific heats near 0°K; x-rays and crystal structure.

**Polytechnic Institute of Brooklyn,** Prof. Erich Hausmann, Brooklyn, N. Y. Apr. 1; M only (Ph.D. in chem., chem. eng. and elec. eng.); no form. 2(2) *graduate fellows*, grading. *R:* optics; electricity; mathematical physics. *U:* spectroscopy; field emission; electrical conduction.

**Princeton University,** Dean of Graduate School, Princeton, N. J. (1–2) *Proctor fellows* (rarely awarded to a first-year student), $1400; (2–4) *fellows*, $700; (1–2) *junior fellows*, $400; (6–8) *research assistants*, $600. Fellows devote time to study and research; assistants give half-time help to some member of staff. All appointees pay $5 f. *R:* line spectra; infra-red spectra; ionization in gases; mass spectrography; nuclear physics; relativistic mechanics; quantum mechanics and its applications to atomic and molecular problems. *U:* spectroscopy; high voltage; mass spectrographs; cyclotron.

**Rensselaer Polytechnic Institute,** Chairman of Graduate Committee, Troy, N. Y. 1–2(1–3) *fellows*, 3 credit hrs/wk service *may* be requested, $700 less $300 t. and f.; ?(?) *scholars*, study and research, $100 to $300, less $300 t. and f. *R:* optics; acoustics; radio and communication; properties of metals.

**Rice Institute,** Prof. H. A. Wilson, Houston, Tex. Apr.1; no form. 2(5) *fellows*, 2 afternoons/wk asst., $500 less $34 f. *R:* nuclear physics; photoelectric conductivity; magnetic properties; etc.

**Rutgers University,** New Brunswick, N. J. 2(4) *graduate assistants*, half-time lab. asst., $600, t. and f. remitted; 2 (varies) *graduate scholars*, t. and f. remitted. *R:* solid state; spectroscopy; atomic constants; optics; radioactivity.

**Smith College,** Northampton, Mass. Ordinarily M only. 1(2) *teaching fellows*, lab. and lec. asst., apply by letter to Prof. A. T. Jones, $600; 1 *fellow*, study and research, apply on form to Dean Marjorie H. Nicolson, stipend consists of board and room. *R:* nuclear physics in connection with an electrostatic generator; sound; electronics; photography; ultraviolet spectrophotometry.

**Stanford University,** Prof. D. L. Webster, Stanford University, Calif. May 1; no form. 1(6) *teaching assistant*, lab. teach., $700 less $130/quarter or $8/unit t. *R:* x-rays; atmospheric electricity; nuclear research; theoretical physics.

**State College of Washington,** Pullman, Wash. M only. 4 *graduate assistants*, max. of 20 hr/wk teach., grading and lab. asst., $450. *R:* work functions of metal surfaces; x-rays; internal friction in metals and alloys; electron optics; theory of solids.

**Syracuse University,** Syracuse, N. Y. M only. 2(6) *graduate assistants*, 12 hr/wk lab. teach., apply to Prof. W. R. Frederickson, $600; 1(2) *scholars*, 3 hr/wk service, apply to Dean W. L. Bray, t. remitted. All appointees pay $5 f. *R:* thermal conduction and expansion; magnetism; electronics; electric conduction in gases; spectroscopy.

**Teachers College, Columbia University,** Prof. S. R. Powers, New York, N. Y. Apply any time; no form. 1(1) *graduate assistant*, physical science lab., grading, educational research, etc., $1000 less $20 f.; (1) *graduate assistant*, physical and biological science lab. and demonstrations, etc., $500 less $12.50 per point t. and $5–$10 f. *R:* science teaching and science education.

**Tufts College,** Prof. J. R. Harrison, Medford, Mass. May 1; M only; no form. 0(2) *assistants*, lab. asst., $1000 less $5 f. *U:* vacuum tubes; piezoelectricity.

**Tulane University,** New Orleans, La. M only; Apr. 15; no form. 4(5) *graduate assistants*, 10 hr/wk lab. teach., $500 less $25 f.; unassigned nonservice *scholarships*, $250. For assistantships apply before Mar. 11 to Prof. D. S. Elliott; for scholarships apply to Graduate Dean. *R:* electronics; photoelectric spectrophotometry; infra-red spectroscopy; x-ray analysis.

**Union College,** Prof. P. I. Wold, Schenectady, N. Y. M only; May 1; no form. 2(2) *graduate assistants*, lab. asst., $600 less $90 t. and f. *R:* properties of metals; vacuum tube circuits.

**University of Akron,** Prof. F. F. Householder, Akron, O. M only; May 1; no form. 2(4) *graduate assistants*, 15 hr/wk lab. asst., grading, $450.

**University of Arizona,** Graduate Dean, Tucson, Ariz. M only. 1(1) *fellow*, lab. asst., $350 less $54 t. and f. *R:* x-ray diffraction patterns; solar radiation (spherical absorber). *U:* astrophysics; photoelectric and photographic photometry with 36-in. reflector.

**University of British Columbia,** Prof. G. M. Shrum, Vancouver, Can. M only. 5–8 *graduate assistants*. *R:* spectroscopy.

**University of California,** Berkeley, Calif. Feb. 20. 10–15(32) *teaching assistants*, lab. asst., grading, etc., $650; various *graduate fellows*, $300 to $1000. All appointees pay $55 f.; nonresident t. of $150 usually remitted. Applicants not already in residence should apply for assistantships. *U:* spectroscopy; nuclear physics; cosmic rays; mobility of ions; mathematical physics.

**University of California at Los Angeles,** Prof. Joseph Kaplan, Los Angeles, Calif. 3(8) *teaching assistants*, lab. teach., $650, apply before Mar. 1; ?(5) *readers*, grading, $25 to $100, apply before Sept. 1. All appointees pay $50 f.; nonresidents also pay $150 t. unless exempted on basis of high scholarship. *R:* acoustics; spectroscopy; meteorology; upper atmosphere. *U:* infra-red spectroscopy; acoustics; upper atmosphere, meteorology.

**University of Chicago,** Committee on Fellowships and Scholarships, Chicago, Ill. 3(6) *graduate assistants*, teach. and lab. asst., $750 less $33.33 f./course; 4(6) *fellows*, lab. asst., grading, etc., $500 less $300 t. and f.; 5(5) *graduate service scholars*, lab. asst., library work, etc., remission of $300 t. and f.

**University of Cincinnati,** Graduate Dean, Cincinnati, O. Mar. 15. 0(1) *graduate assistant*, lab. asst., $400; 6(5–6) *fellows*, study and research, $100 to $500; 6 *scholars*, study and research, remission of various f. All appointees pay $5 f. *R:* spectroscopy; soft x-rays; atmospheric electricity. *U:* soft x-rays, etc.

**University of Denver,** Dean A. C. Nelson, Denver, Colo. M only; Apr. 1. 1–2(2) *graduate assistants*, lab. and research asst., $125 to $250 less $234 t. and f. *U:* high altitude laboratory (14,000 ft); cosmic rays.

**University of Idaho,** Prof. G. W. Hammar, Moscow, Idaho. Apr. 1; M only; no form. 1(2) *teaching fellow*, teach. and lab. asst., $500. *R:* various fields. *U:* optical spectroscopy.

**University of Illinois,** Urbana, Ill. 6(23) *graduate assistants*, half-time teach., $700; *scholars* and *fellows*, study and research, $300 to $600. New students pay $10 f. For assistantships apply before Feb. 15 to Prof. F. W. Loomis; for other appointments apply before Feb. 15 to Dean R. D. Carmichael. *R:* spectroscopy (band and line spectra, hyperfine structure); nuclear physics; mass spectra; acoustics; theoretical physics; fluorescence; electron optics. *U:* spectroscopy (vacuum, visible, near infra-red); nuclear physics (cyclotron, linear accelerator, Ra-Be source); acoustics; fluorescence; theoretical physics.

**University of Kansas,** Lawrence, Kan. ?(2–3) *graduate assistants*, 20 hr/wk class and lab. teach. and papers, $450; *university scholars* (8–9 for whole university), 0–6 hr/wk service, $250; *university fellows* (3 for whole university), 0–6 hr/wk service, $400. All appointees pay $70 and lab. f.; nonresidents pay additional $110. For assistantships, apply any time by letter to Prof. F. E. Kester; for other appointments apply on form to Dean E. B. Stouffer. *U:* dielectric constants of liquids and gases; thermal and optical properties of liquid alloys.

**University of Kentucky,** Prof. Wm. S. Webb, Lexington, Ky. May 1. 6(8) *graduate assistants*, 10 hr/wk lab. asst., $500 less $100 t. for state residents or $160 for nonresidents. For information concerning *fellows* and *scholars*, address Graduate Dean.

**University of Louisville,** Louisville, Ky. M only. No graduate appointments available at present time.

**University of Maine,** Prof. C. E. Bennett, Orono, Me. Apply any time; M only; no form. 2(2) *graduate assistants*, lab. asst., $600.

**University of Manitoba,** Winnipeg, Manitoba, Can. M only. No appointments available in 1941–42. *R:* biophysics; thermionics; x-rays; radioactivity; spectroscopy. *U:* Cancer research laboratory, with 1 gm of radium, half in needles and half in solution.

**University of Maryland,** Prof. Chas. G. Eichlin, College Park, Md. M only; June 1; no form. 2 *fellows*, 9 hr/wk lab. teach., $400.

**University of Michigan,** Ann Arbor, Mich. 5 (15) *teaching assistants*, lab. teach., $300 to $700; 3(1–6) *research assistants*, research asst., $100 to $1200; 2(1–3) *fellows*, study and research, $500 to $1000. All appointees pay $120 t. if state resident, $200 if nonresident. For assistantships apply to Prof. R. A. Sawyer by Mar. 15; for fellowships, to Graduate Dean by Feb. 15. *R:* sound; spectroscopy; x-rays; electronics; vacuum tubes; radio. *U:* infra-red spectroscopy; nuclear physics; cyclotron; high potential.

**University of Minnesota,** Minneapolis, Minn. 5(15) *teaching fellows*, lab. teach., $600. *R:* x-rays; electronics; atomic structure; spectroscopy; biophysics. *U:* nuclear physics; mass spectroscopy.

**University of Mississippi,** Dean D. H. Bishop, University, Miss. M only; July 1. 1(1) *graduate scholar*, teach. and lab. asst., $300 less $50.75 t. and f. for state residents or $100.75 for nonresidents. Applicants interested in a particular research field should describe their preparation for it.

**University of Missouri,** Columbia, Mo. 1(4) *graduate assistants*, lab. teach., $500 less lab. f. and $30–$60 t. and f.; *Gregory scholars* (10 for university), study and research, $300 less lab. f. and $60 t. and f. An advanced student who has begun research may be awarded a $600 fellowship for study and research. For assistantships apply by letter to Prof. H. M. Reese; for scholarships, apply on special form to Graduate Dean. *R:* x-rays; biophysics; photovoltaic effect; ionizations. *U:* diffraction of x-rays in liquids; biological effects of radiation.

**University of Nebraska,** Prof. H. H. Marvin, Lincoln, Neb. M only. 4(5) *graduate assistants*, 6–8 hr/wk, in charge of lab. secs., $480 less $6 f. *R:* spectroscopy; x-rays; conduction of electricity in gases; electric and magnetic measurements.

**University of New Mexico.** M only; last date for application indefinite. 1(1) *graduate fellow*, lab. teach., $400 less $62 t. and f. *R:* geophysics. *U:* lightning and thunderstorm phenomena.

**University of North Carolina,** Dean W. W. Pierson, Chapel Hill, N. C. Feb. 15. 1(1) *teaching fellow*, usually awarded for third or fourth graduate year, $500; 5 *graduate assistants*, $450. Total service for each appointee is 16 hr/wk lab. asst. and grading. One-half graduate assistantships may be assigned to suitable applicants needing financial aid. All appointees pay $26 f. which covers laundry, infirmary, etc. *R:* infra-red spectroscopy; biophysics; interactions of electrons and $\gamma$-rays with matter. *U:* excellent grating spectrographs for infra-red; cloud chambers.

**University of Notre Dame,** Secretary of Graduate School, Notre Dame, Ind. 2(12) *graduate assistants*, lab. asst., $750 less $10/sem hr t. *R:* physics of rubber; theoretical physics (solid state; nuclear physics); electronics (interaction of low energy electrons with matter); nuclear physics (interaction of high energy electrons with matter). *U:* electronics (surface properties of metallic and nonmetallic solids); nuclear physics (1.8-Mev Van de Graaff electrostatic generator in operation; large pressure electrostatic generator under construction).

**University of Oklahoma,** Prof. H. L. Dodge, Norman, Okla. Mar. 1 if possible, 1(3) *industrial fellows*, 20 hr/wk, $400 to $750; 4(6) *graduate assistants*, 18 hr/wk lab. asst., $400; 1(3) *instructor* (*half-time*), teaching college physics, $800; 2(2) *research fellows and scholars*, 12 hr/wk on own research, $300; 3(4) *university scholars*, t. and f. remitted. *R:* Raman effect, infra-red absorption spectra; molecular and crystal structure; geophysics; photoelectricity; x-ray spectroscopy; separation of isotopes. *U:* Raman effect.

**University of Pennsylvania,** Philadelphia, Pa. 1(2) *laboratory assistant*, half-time lab. asst., $500, apply by letter to Physics Department. For the following appointments, which are open to general competition in the graduate school and require no services, apply on form to Dean E. B. Williams: *Harrison research fellows*, $1500; *Harrison fellows*, $600; *Harrison* and *university scholars*, $200 down to t. remitted; several fellowships supported by industrial laboratories, approx. $600. All appointees pay minor fees. *U:* the solid state; x-ray and electron diffraction; radioactivity; nuclear physics.

**University of Pittsburgh,** Prof. E. Hutchisson, Pittsburgh, Pa. 5(11) *graduate assistants*, lab. teach., $500. *R:* not limited. *U:* physics of metals; optical pyrometry; nuclear physics; seismology.

**University of Rochester,** Prof. L. A. DuBridge, Rochester, N. Y. 2(7) *graduate assistants*, lab. asst., $600; 1(4) *fellow*, research asst., research experience required, $600; 1(4) *graduate assistant*, Institute of Optics, teach. and research (apply to Prof. Brian O'Brien), average stipend, $600. All appointees pay $5 f. *R:* electron emission; nuclear physics; optics; biophysics. *U:* cyclotron; optics.

**University of Southern California,** Mrs. F. B. Watt, Los Angeles, Calif. 1(2) *fellow*, lab. asst., $500 less $240 t. and f. *R:* electrical measurements; vacuum-tube phenomena.

**University of Utah,** Salt Lake City, Utah. M only. 3(5) *teaching fellows*, 15 hr/wk lab. asst., $300 less t. and f.

**University of Vermont,** Prof. G. P. Burns, Burlington, Vt. M only; Feb. 1. 4(8) for university, *research fellows*, max. of 8 hr/wk lab. asst., $800. *U:* photoelectric and thermoelectric effects; x-rays.

**University of Virginia,** University Station, Charlottesville, Va. 1(5) *service fellows*, lab. and quiz sections, $530; 1(2) *service fellow*, lab. and quiz sections, $380; 2(2) *Philip Francis du Pont junior fellows*, $200–$275; 1(3) *Philip Francis du Pont senior fellow*, $300–$375; 2(2) *Philip Francis du Pont research fellows*, $475–$830. For service fellowships apply to Prof. L. G. Hoxton; for duPont fellowships,which involve research and study only, apply to Secretary of Graduate Dean. *U:* ultracentrifuge and electric discharge problems.

**University of Wisconsin,** Madison, Wis. Feb. 15. 2–3(7–8) *research assistants*, 20 hr/wk service, $600; 4–5(15–16) *teaching assistants*, 14 hr/wk lab. teach., $750; 0–1(1–2) *fellow*, 4 hr/wk service, $600; 3–4(4–5) *alumni research fellows* and *scholars*, study and research, $400 to $600; 0–1(0–1) *scholar*, study and research, $250. All appointees pay $65 t. and f. For assistantships apply to Prof. L. R. Ingersoll; for fellowships and scholarships, to Dean E. B. Fred. *U:* nuclear physics; electronics; spectroscopy; thermodynamics.

**University of Wyoming,** Laramie, Wyo. M only; no form. 1(1) *graduate assistant*, lab. and quiz asst., grading, $500 less $30 t. and f. *R:* single crystals (zinc); evaporation.

**Utah State Agricultural College,** Logan, Utah. Apr. 1; M only. 1(2) *graduate teaching assistantship*, $400. *R:* soil physics, linear resonance accelerator, cosmic rays.

**Vanderbilt University,** Graduate Dean, Nashville, Tenn. M only. 1(1) *scholar*, 6 hr/wk library, $300 less $203 t. and f.; 2(2) *fellows*, 6 hr/wk lab. asst., $500 less $203 t. and f.

**Wellesley College,** Prof. Louise S. McDowell, Wellesley, Mass. M only; no form. 1(2) *assistant*, 22 hr/wk lab. teach., etc., $600. *R:* dielectrics.

**Wesleyan Univeristy,** Department Chairman, Middletown, Conn. Apply any time; M only; no form. 1(3–5) *half-time graduate assistants*, lab. asst., grading, etc., $600 to $800. *R:* piezoelectricity; high frequency phenomena.

**Western Reserve University,** Prof. H. W. Mountcastle, Cleveland, O. M only. 1 *graduate assistant*. *R:* supersonics, etc.

**Westinghouse Electric and Manufacturing Co.,** J. H. Belknap, Technical Employment and Training Dept., Pittsburgh, Pa., Feb. 15. 5 *post-doctorate research fellows*, complete freedom and support of research on project proposed by applicant, the work to be carried on in Westinghouse Research Laboratories, $2400. *U:* nuclear physics; electronics; ferromagnetism; dielectrics; chemical physics; physical metallurgy; applied mechanics.

**Worcester Polytechnic Institute,** Worcester, Mass. No appointments open for 1941–42.

**Yale University.** New Haven, Conn. 3(8) *laboratory assistants*, lab. asst., grading, $800; 2(2) *research assistants*, lab. asst., $1200 to $1500; *scholars* and *fellows*, study and research, $315 to $1150. All appointees pay $315 t. and f. For assistantships apply (no form) to Prof. W. W. Watson; for fellowships and scholarships obtain form from Graduate Dean. *U:* magnetism; spectroscopy; radioactivity; x-rays; nuclear physics; cosmic rays; low temperatures; theoretical physics.

[*This list will be continued in the February, 1941 issue.*]

# RECENT PUBLICATIONS

## Textbooks and References for General College Physics

**College Physics.** John A. Eldridge, Professor of Physics, University of Iowa. Ed. 2. 714 p., many figs., 53 tables, 14×22 cm. *Wiley*, $3.75. In the Preface of this revised and enlarged edition of a well-known textbook, Professor Eldridge says:

> As I write this my radio brings words from the chief of the U. S. Navy, advising young men about training for their country's need: "Young man, learn about machines; become acquainted with the operation of your automobile, of your radio." Educational ideas have been changed overnight. Yesterday the dominant idea was untrammeled self-expression; today the word discipline has come back into the educational vocabulary. Discipline and training. If we believe all we hear there is to be a new emphasis upon the physical sciences . . . it would certainly seem that college students, future leaders of democracy, must learn to think—to think harder, more objectively, more incisively, more in anticipation of the future event—or we shall all be told not to think at all.

The first edition of this book received an extensive and favorable review in this journal [6, 222 (1938)].

**Light and Colour in the Open Air.** M. Minnaert, Professor at the University of Utrecht. Tr. by H. M. Kremer-Priest; rev. by K. E. Brian Jay. 274 p., 160 figs., 16 plates, 14×22 cm. *G. Bell & Sons*, 15 s. net. The author of this unique and useful collection of 233 "notes on physical nature study" points out that a book on all that is to be seen of particular interest to the student of physical science in inanimate nature should be just as necessary to him as a book on flora and fauna is to the biologist. In the teaching of physics we have not sufficiently realized the importance of outdoor observations. "They help us in our increasing efforts to adapt our education to the requirements of everyday life; they lead us by natural methods to ask a thousand questions, and, thanks to them, we find later on that what we learnt at school is to be found again and again beyond our school walls." The notes comprising this book have been collected by the author over a period of many years. In many of them he raises interesting questions or gives references to the literature that serve to stimulate further observation and thought. The 13 chapters into which the notes are grouped have titles such as "Reflection of light," "After-images and contrast phenomena," "Judging shape and motion," "Light and colour of the landscape and of the sky," "Luminous plants, animals and stones." In the last-named chapter, for example, one finds notes on glow-worms, phosphorescence of the sea, luminous wood and leaves, cats' eyes at night, reflection of light on mosses, fluorescence of plant juices, phosphorescent ice and snow, scintillations from stones, and will-o'-the-wisps. Many of these are things of everyday life, here made interesting and meaningful because they are placed in a scientific framework; but many others are things unfamiliar to the average person, although they can be seen almost any moment, once attention has been drawn to them. All can be observed without the aid of special instruments or technics, although they are rendered immensely more interesting and significant to the reader who is already somewhat familiar with the underlying physical principles. The book should be valuable as collateral reading for beginners, as well as providing a rich source of illustrative material for the teacher.

## Intermediate Textbooks

**Light.** F. Bray, Late Science Master at Clifton College. Ed. 2. 381 p., 253 figs. and plates, 12×18 cm. *Longmans, Green*, $2.50. The grade of this textbook is semi-intermediate. Calculus is employed in a few sections which can, however, be omitted. In the present edition, the chapters on photometry and on spectra have been rewritten; and the sign convention now adopted is to regard real objects and images as being formed in positive spaces, and virtual objects and images, in negative spaces.

**Sound.** E. G. Richardson, Senior Lecturer in Physics, King's College, Newcastle-on-Tyne, University of Durham; Visiting Lecturer in Applied Acoustics at the University Colleges of London and Dublin. Ed. 3. 346 p., 118 figs., 13×21 cm. *Longmans, Green*, $5.25. The third edition of this useful textbook differs from the second (1935) mainly in that the chapters on impedance, supersonics and sound reproduction have been brought up to date and extended. As in the earlier editions, more than ordinary attention is given to the experimental researches in the field and to musical, physiological and technologic applications. The specific references to the literature are numerous for a book of intermediate grade. Elementary calculus is employed.

## Advanced Textbooks and Treatises

**Properties of Ordinary Water-Substance.** N. Ernest Dorsey, Physicist, National Bureau of Standards. 697 p., 289 tables, 12 figs., 15×23 cm. *Reinhold*, $15. In this important compilation of the properties of pure, ordinary water in all its phases, in its synthesis and dissociation, and in its transition from phase to phase, the author has made a great effort to present, either actually or by reference to the literature, all the existing material likely to be of interest to the worker in this field. All the data pertaining to the subject in the *International Critical Tables* are here assembled in a revised, extended and more accessible form, and types of material are now included that, for one reason or another, were omitted from the aforementioned tables. Nor is the compilation confined to numerical data, for there is much descriptive information, including discussions of the significance and derivation of data and

formulas. The units for the numerical data are in every case specified clearly, in the best physical manner. An extensive index is provided. Although no attempt is made in general to include information on the behavior of water in the presence of another substance, data are given on the effects of the presence of air, the solubilities and diffusivities of certain gases in water, and other similar effects that are incidental to an understanding of the behavior of water itself. This work is No. 81 in the *American Chemical Society Series of Scientific and Technologic Monographs*.

**The Mathematical Papers of Sir William Rowan Hamilton. Vol. II, Dynamics.** Edited for the Royal Irish Academy by A. W. CONWAY, Professor of Mathematical Physics, University College, Dublin, AND A. J. MCCONNELL, Fellow and University Professor of Natural Philosophy, Trinity College, Dublin. 671 p., 22×30 cm. *Cambridge Univ. Press* and *Macmillan*, $19.50. HAMILTON'S published works alone might well have formed the entire output of a long and laborious life; yet they represent only a small part of his extensive investigations, as the latter are revealed in the enormous collection of manuscripts in the possession of Trinity College, Dublin. HAMILTON was a precise and meticulous writer, and the material which he did publish was carefully rewritten by him in the form of compact and condensed abstracts, which on this account are often difficult to read and comprehend. The present volume is confined to the papers on the dynamics of bodies and of light, and on the "Calculus of principal relations," which is a natural extension of HAMILTON'S method of the principal function to obtain integrals of total and even partial differential equations. About three-quarters of the material appears in print for the first time, and this includes all the important papers in the volume except the two famous essays that have been of such great value in theoretical physics and particularly in quantum theory; namely, those entitled "On a general method in dynamics" (1834–1835), in which HAMILTON develops all theoretical dynamics by the aid of one central function, introduces the principal function and its properties, and establishes his equations of motion. Much of the hitherto unpublished material is in an unfinished condition, for it was not intended for publication; but it is just this preliminary work, often involving the solution of many specific problems in a laborious transition from particulars to general conclusions, that shows most clearly HAMILTON'S method of working and his great power of manipulation. The book is carefully edited and includes 37 pages of editorial notes. It is beautifully printed. Volume I, previously published, contains HAMILTON'S papers on geometrical optics.

**Electrodynamics.** LEIGH PAGE and N. I. ADAMS, Yale University. 518 p., 115 figs., 15×23 cm. *Van Nostrand*, $6.50. This advanced textbook in theoretical electrodynamics is a fitting sequel to the authors' volume, *The Principles of Electricity* and to the senior author's textbook, *Introduction to Theoretical Physics*, both of which appeared some years ago in this series. The relativistic attitude is adopted from the start and the Maxwell equations are deduced on the basis of the emission theory. Because of limitations of space, the authors have omitted electrostatics and magnetostatics and refer the reader to other readily available texts. The first 128 pages are devoted to a review of three-dimensional vector analysis, a comprehensive treatment of dyadics and a development of all the relativistic theorems required in the ensuing pages. Then proceeding to their main objective, the authors derive the equations of the electromagnetic field from a consideration of a point charge and with the aid of retarded quantities, and, after stating the necessary and sufficient boundary conditions, generalize these equations to the case of any number of such charges. The fields of the elementary charge moving with constant velocity and with constant acceleration are then considered in relation to the force equation, and application is made to the spinning electron both at rest and in motion in order to deduce the equation of motion of the spinning electron. The electromagnetic equations are next applied to material mediums, and the energy, stress and momentum relations are obtained to study electromagnetic waves in homogeneous isotropic mediums. Considerable space is devoted to radiation and to radiating systems, including a point charge, a group of point charges and the spinning electron. This is followed by a discussion of free and forced oscillations in conductors of various shapes. The electromagnetic theory of light is then developed and applied. Finally there is a four-dimensional formulation of the equations of electromagnetism with an extensive presentation of four-dimensional vector and tensor analysis and an application of general dynamical methods to circuit theory, the magnetron and cosmic-ray trajectories. This book should serve not only as an excellent text for a graduate course but also as a valuable reference in which may be found a lucid and logical presentation of electrodynamics in the light of recent developments. The publishers are also to be commended on their workmanship which maintains the high standard of the other books in this series.—L. G. P.

## CHEMICAL PHYSICS AND PHYSICAL CHEMISTRY

**Electrocapillarity—the Chemistry and Physics of Electrodes and Other Charged Surfaces.** J. A. V. BUTLER, Lecturer in Chemistry, University of Edinburgh 216 p., 57 fig., 30 tables, 14×22 cm. *Chemical Publishing Co.*, $5. The emphasis in this book is on the work on electrocapillarity that has been done during the last 15 years and with which the author has been closely associated. Particular attention is given to topics that are inadequately discussed in the existing literature. The main subjects treated include seat of the emf in the galvanic cell, thermodynamics of electrode potentials, mechanism and energetics of reversible electrode potentials, electric double layers, electrokinetic phenomena, electrode reactions and overvoltage, concentration polarization and the deposition of metals, various electrode processes. Topics that are omitted or barely mentioned because adequate reviews exist elsewhere include uses for reversible electrodes in determining the thermodynamic properties of electrolytes, polarographic studies, phase boundary potentials between two liquids, and corrosion of metals.

# DIGEST OF PERIODICAL LITERATURE

## Apparatus and Demonstrations

**Notes on the demonstration of "wetter" water.** C. R. Caryl; *J. Chem. Ed.* **17**, 468 (1940). The remarkable reduction in surface tension produced by modern synthetic wetting agents may easily be shown as follows: (1) Powdered sulfur will float indefinitely on the surface of water in a beaker but will sink immediately when a few drops of Aerosol OT (the di-octyl ester of sodium sulfosuccinate) are added to the surface by means of a stirring rod or eyedropper. (2) Into one beaker containing water and another containing water plus 0.1 percent Aerosol OT, powdered sulfur is poured from both hands; the sulfur does not penetrate the surface of the pure water but pours like sand into the beaker of "wetter" water. Pieces of ordinary string, or wads of raw wool or cotton, float in the one beaker but immediately wet and sink in the other. (3) A gram of Aersol OT pressed into the stern of a toy ship, made preferably of balsa wood, will propel the ship back and forth in a tank of water for hours; the speed increases with the temperature of the water.—D. R.

## Editorial Policies of a Scientific Periodical

**On scholarly writing and critical reviewing.** W. L. McAtee; *Sci. Mo.* **51**, 77–79 (1940). The following extracts from this article describe positive points of view to which the *American Journal of Physics*, in its editorial policy, subscribes completely:

"Institutional editing, in general, is well organized and efficient, giving a good degree of dignity, accuracy and clarity to its products. The less formal and mostly unpaid editing of scientific periodicals, however, is often not attended with such favorable results. In a good many instances, apparently, no real editing is done. A few commas may be added or tables and bibliographies made to conform to the adopted style, but on the whole, manuscripts are merely accepted or rejected. This practice evades editorial responsibility and is unfair in that a well-written paper may be accepted, though deficient in accuracy, while a 'diamond in the rough' so far as substance is concerned, may be rejected. To contribute to the advance of knowledge, it is just the authors of these promising but poorly expressed articles who should be helped. Editorial efforts spent in aiding these writers with references, supplementary information and suggestions as to better expression will in most cases be appreciated and, in the long run, will be repaid a thousandfold in terms of improved scholarship.

"To an old-fashioned critic, it appears nowadays that no matter how trivial the topic nor how ill-judged the content of an article, if it be presented in 'constructive' form as an alleged contribution to knowledge, it is accepted for printing. On the other hand, a review calling attention to crudity, incompleteness or illogicality of a published paper is rejected as being 'destructive.'

"Limitation of discussion is not in accord with the spirit of science, in which all questions should be forever open. Suppression is alien to science and should be also to publishing mediums and institutions dealing with any phase of science. Suppression is a trait of authoritarianism which the proletarians usually get for their pains instead of their vaunted brotherhood. Critical writing is equivalent to discussion at meetings. That is usually allowed and is sometimes as liberally provided for as are formal contributions themselves. A meeting at which discussion is not permitted is generally regarded as unsatisfactory, and the parallel situation in publication is equally so.

"Even if other disciplines reject them, science should preserve and encourage its critics. A true scientist is a born sceptic, so why after retaining that quality long enough to make progress in science should he discard it? Scepticism, wholesome doubt, is among his most useful tools and its manifestations in a critic should be welcomed, not condemned. Humanity tends toward mob psychology, but the scientist should remain himself, poised, observant, critical of all things. To become one of a mob is to desert science. Suppression of criticism, rooted in the fatally easy preference for ease and freedom from every irritation or annoyance, is such a desertion and is, in the present state of philosophy, desertion in the face of the enemy."—D. R.

## Check List of Periodical Literature

**Utilizing sun rays.** C. G. Abbot; *Sci. Mo.* **51**, 195–200 (1940). When it becomes possible to produce existing solar power devices in quantities, solar energy probably can be made available, with fair return on the investment, at a maximum cost of 0.5 ct/hp hr.

**Milestones in meteorology.** W. H. Wenstrom; *Sci. Mo.* **51**, 226–232 (1940). A brief history.

**The responsibility of education to society.** H. E. Barnes; *Sci. Mo.* **51**, 248–260 (1940). Educators might as well understand, once and for all, that we cannot save democracy through education, unless we are willing to teach in our schools the materials essential to the salvation of democracy.

**Culture: a scientist's ideal.** R. King; *Sci. Mo.* **51**, 261–268 (1940). A physicist's conception of self-culture.

**Scientific progress and the evolution of capitalism.** G. W. Edwards; *Sci. Mo.* **51**, 65–73 (1940). How will the progress of science be affected by the rapid changes now occurring in the institutions of capitalism?

**Basic principles of the new mechanics.** G. Gamow; *Sci. Mo.* **51**, 358 (1940). A popular account.

**A scientific approach to religion.** J. S. O'Conor; *Sci. Mo.* **51**, 368–372 (1940). The author of this stimulating article is professor of physics in Georgetown University.

# Author Index to Volume 8

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